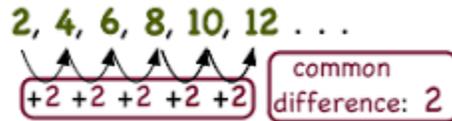


Revision Notes on Arithmetic Progressions

Arithmetic Progressions

An **Arithmetic Progression** is a sequence of numbers in which we get each term by adding a particular number to the previous term, except the first term.



- Each number in the sequence is known as **term**.
- The fixed number i.e. the difference between each term with its preceding term is known as **common difference**. It can be positive, negative or zero. It is represented as 'd'.

Some Examples of Arithmetic Progressions

Common difference	Value of d	Example
$d > 0$, positive	10	20, 30, 40, 50,...
$d < 0$, negative	-25	100, 75, 50, 25, 0
$d = 0$, zero	0	5, 5, 5, 5,...

General form of Arithmetic Progression

$$a, a + d, a + 2d, a + 3d, \dots$$

Where the first term is 'a' and the common difference is 'd'.

Example

Given sequence is 2, 5, 8, 11, 14,...

Here, $a = 2$ and $d = 3$

$$d = 5 - 2 = 8 - 5 = 11 - 8 = 3$$

First term is $a = 2$

$$\text{Second term is } a + d = 2 + 3 = 5$$

Third term is $a + 2d = 2 + 6 = 8$ and so on.

Finite or Infinite Arithmetic Progressions

1. Finite Arithmetic Progression

If there are only a limited number of terms in the sequence then it is known as **finite** Arithmetic Progression.

$$229, 329, 429, 529, 629$$

2. Infinite Arithmetic Progression

If there are an infinite number of terms in the sequence then it is known as **infinite** Arithmetic Progression.

2, 4, 6, 8, 10, 12, 14, 16, 18.....

The n^{th} term of an Arithmetic Progression

If a_n is the n^{th} term, a_1 is the first term, n is the number of terms in the sequence and d is a common difference then the n^{th} term of an Arithmetic Progression will be

$$a_n = a_1 + (n - 1)d$$

The diagram shows the formula $a_n = a_1 + (n - 1)d$ with four colored arrows pointing upwards to each term. Below each arrow is a text label: a green arrow points to a_n with the label " n^{th} term in the sequence"; a blue arrow points to a_1 with the label "1st term in the sequence"; an orange arrow points to $(n - 1)$ with the label "number of terms in the sequence"; and a red arrow points to d with the label "common difference".

Example

Find the 11th term of the AP: 24, 20, 16,...

Solution

Given $a = 24$, $n = 11$, $d = 20 - 24 = -4$

$$a_n = a + (n - 1)d$$

$$a_{11} = 24 + (11 - 1) - 4$$

$$= 24 + (10) - 4$$

$$= 24 - 40$$

$$= -16$$

Arithmetic Series

The arithmetic series is the sum of all the terms of the arithmetic sequence.

The arithmetic series is in the form of

$$\{a + (a + d) + (a + 2d) + (a + 3d) + \dots\}$$

Sum of first n terms of an Arithmetic series

Sum of the first n terms of the sequence is calculated by

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

Example

If Radha save some money every month in her piggy bank, then how much money will be there in her piggy bank after 12 months, if the money is in the sequence of 100, 150, 200, 250,respectively?

Solution

Given sequence is-

100, 150, 200, 250, ...

$a = 100$ (first term)

$d = 50$ (common difference)

$n = 12$ (as we have to calculate money of 12 months)

Now we will put the values in the formula

$$\begin{aligned}S_n &= \frac{n}{2} [2a + (n - 1)d] \\&= \frac{12}{2} [2(100) + (12 - 1)50] \\&= \frac{12}{2} [200 + 550] \\&= \frac{12}{2} (750) \\&= 6(750) \\&= 4500\end{aligned}$$

So the money collected in her bank in 12 months is Rs. 4500.

But when we have **finite Arithmetic Progression** or we know the last term of the sequence then the sum of all the given terms of the progression will be calculated by

$$S_n = \frac{n}{2}[a + I]$$

Where $I = a + (n - 1)d$ i.e. the last term of the finite Arithmetic Progression.

Remark: The sum of the infinite arithmetic sequence does not exist.

Example

Find the sum of the sequence 38, 36, 34, 32, 30.

Solution

Given

$$a = 38, l = 30 \text{ and } n = 5$$

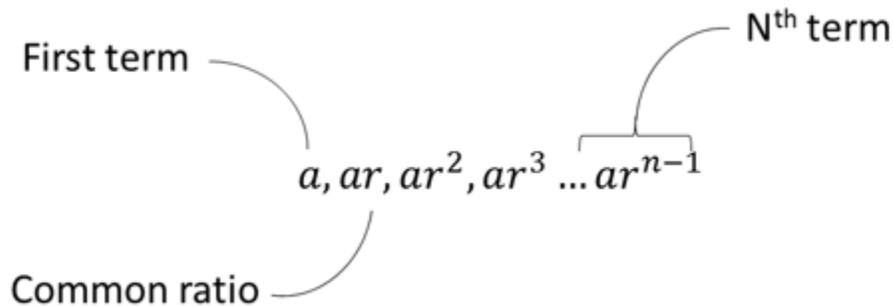
$$S_n = \frac{n}{2}(a + l)$$

$$\begin{aligned} S_5 &= \frac{5}{2}(38 + 30) \\ &= \frac{5}{2}(68) \\ &= 170 \end{aligned}$$

Geometric Progression

A **Geometric Progression** is a sequence of numbers in which we get each term by multiplying or dividing a particular number to the previous term, except the first term.

The ratio between every term to the next term is constant.



n^{th} term of the Geometric Sequence

$$a_n = a_1 r^{n-1}$$

Sum of Geometric Series

Finite series	$S_n = \frac{a_1(1 - r^n)}{1 - r}$
Infinite series with $-1 < r < 1$	$S_\infty = \frac{a_1}{1 - r}$
Infinite series with $r \geq 1$	Sum does not exist.

Harmonic Progression

It is the reverse of Arithmetic Progression. If $a, a + d, a + 2d, \dots$ is an Arithmetic Progression then the harmonic progression is

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots$$

n^{th} term of Harmonic Progression

$$T_n \text{ of H.P.} = \frac{1}{T_n \text{ of A.P.}}$$

Remark: There is no special formula for finding the sum of the harmonic series so we can calculate the sum of the arithmetic series and then take the reciprocal of it which will be the sum of the harmonic series.

The harmonic series is divergent to infinity.

Arithmetic Mean

Arithmetic mean is the average of the two numbers. If a, b and c are in Arithmetic Progression then the arithmetic mean of a and c will be

$$b = \frac{(a + c)}{2}$$

Some Important Points

$$S_n = \frac{n(n+1)}{2}$$

- Sum of first n positive integers is given by
- The difference between the sum of the first n terms and first $(n - 1)$ terms is also the n^{th} term of the given Arithmetic Progression.

$$a_n = S_n - S_{n-1}$$

Geometric Mean

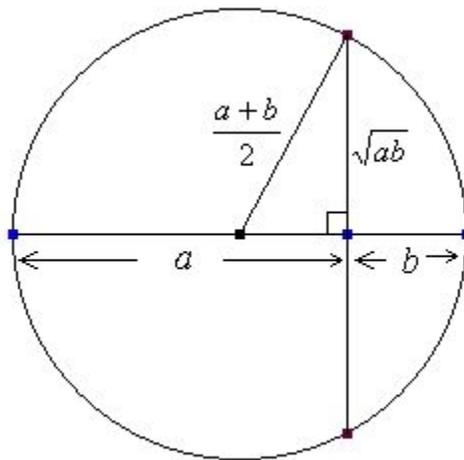
Geometric mean is the average of two numbers. If a and b are the two numbers then the geometric mean will be

$$GM = \sqrt{ab}$$

Relationship between A.M. and G.M.

As we have seen above the formula for the Arithmetic mean and the Geometric mean are as follows-

$$A.M. = \frac{a + b}{2} \text{ and } GM = \sqrt{ab}$$



Where a and b are the two given positive numbers.

Let A and G be A.M. and G.M.

So

$$A = \frac{a + b}{2} \text{ and } G = \sqrt{ab}$$

Now let's subtract the two means with each other

$$\begin{aligned} A - G &= \frac{a + b}{2} - \sqrt{ab} \\ &= \frac{a + b - 2\sqrt{ab}}{2} \\ &= \frac{(\sqrt{a} - \sqrt{b})^2}{2} \geq 0 \end{aligned}$$

This shows that $A \geq G$

Properties of relationship of A.M and G.M.

Property I: If the Arithmetic Mean and Geometric Mean of two positive numbers a and b are A and G respectively, then

$A > G$

As $A - G > 0$

Property II: If A be the Arithmetic Mean and G be the Geometric Mean between two positive numbers a and b, then the quadratic equation whose roots are a, b is

$$x^2 - 2Ax + G^2 = 0$$

Property III: If A is the Arithmetic Means and G be the Geometric Means between two positive numbers, and then the numbers are

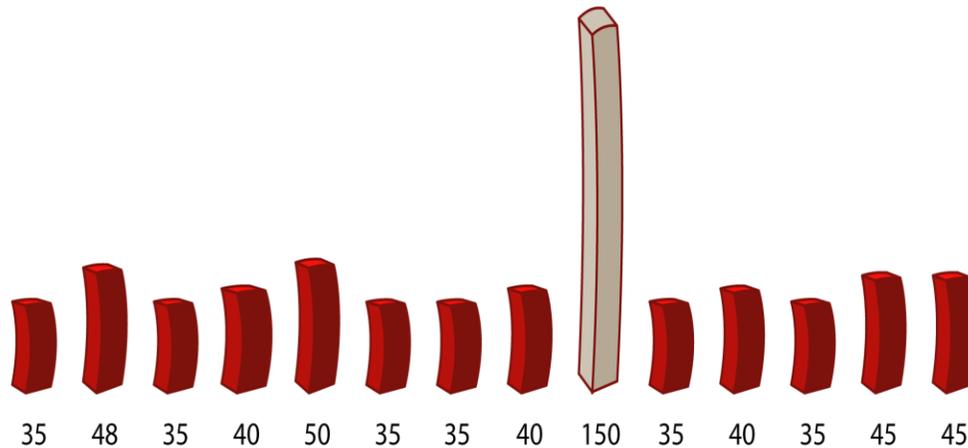
$$A \pm \sqrt{A^2 - G^2}$$

Harmonic Mean

Harmonic mean is calculated by dividing the number of items with the sum of the reciprocals of all the items.

$$\text{Harmonic Mean} = \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right)}$$

Harmonic mean is used when there are extreme observations. Like



In this case if we will use arithmetic mean to find the average then we will get the inaccurate average.

Relation between AM, GM and HM

If a and b are two positive real numbers then

$$AM = \frac{a+b}{2}$$

$$GM = \sqrt{ab}$$

$$HM = \frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b} = \frac{GM^2}{AM}$$

$$(\sqrt{a} - \sqrt{b})^2 \geq 0$$

$$a + b - 2\sqrt{a}\sqrt{b} \geq 0$$

$$a + b \geq 2\sqrt{a}\sqrt{b}$$

$$\frac{a+b}{2} \geq \sqrt{ab}$$

$$AM \geq GM$$

$$a + b \geq 2\sqrt{a}\sqrt{b}$$

$$(a + b)\sqrt{ab} \geq 2\sqrt{ab}\sqrt{ab}$$

$$(a + b)\sqrt{ab} \geq 2ab$$

$$\sqrt{ab} \geq \frac{2ab}{a+b}$$

GM \geq HM

The combination of above two inequalities shows that

AM \geq GM \geq HM

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