Revision Notes on Quadratic Equations

Quadratic Polynomial

A polynomial, whose degree is 2, is called a quadratic polynomial. It is in the form of

$p(x) = ax^2 + bx + c$, where $a \neq 0$

Quadratic Equation

When we equate the quadratic polynomial to zero then it is called a Quadratic Equation i.e. if

$p(x) = 0$, then it is known as Quadratic Equation.

Standard form of Quadratic Equation

$$ax^2 + bx + c = 0$$

where a, b, c are the real numbers and $a \neq 0$

Types of Quadratic Equations

1. Complete Quadratic Equation $ax^2 + bx + c = 0$, where $a \neq 0$, $b \neq 0$, $c \neq 0$

2. Pure Quadratic Equation $ax^2 = 0$, where $a \neq 0$, $b = 0$, $c = 0$

Roots of a Quadratic Equation

Let $x = \alpha$ where α is a real number. If α satisfies the Quadratic Equation $ax^2 + bx + c = 0$ such that $a\alpha^2 + b\alpha + c = 0$, then **$\alpha$ is the root of the Quadratic Equation**.

As quadratic polynomials have degree 2, therefore Quadratic Equations can have two roots. So the zeros of quadratic polynomial $p(x) = ax^2+bx+c$ is same as the roots of the Quadratic Equation $ax^2 + bx + c = 0$.

Methods to solve the Quadratic Equations

There are three methods to solve the Quadratic Equations-

1. Factorisation Method

In this method, we factorise the equation into two linear factors and equate each factor to zero to find the roots of the given equation.

Step 1: Given Quadratic Equation in the form of $ax^2 + bx + c = 0$.

Step 2: Split the middle term bx as mx + nx so that the sum of m and n is equal to b and the product of m and n is equal to **ac**.

Step 3: By factorization we get the two linear factors (x + p) and (x + q)

$ax^2 + bx + c = 0 = (x + p)(x + q) = 0$

Step 4: Now we have to equate each factor to zero to find the value of x.

$$x^2 - 2x - 15 = 0$$

$$(x + 3)(x - 5) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 5 = 0$$

$$x = -3 \quad \text{or} \quad x = 5$$

$$x = \{-3, 5\}$$

These values of x are the two roots of the given Quadratic Equation.

2. Completing the square method

In this method, we convert the equation in the square form $(x + a)^2 - b^2 = 0$ to find the roots.

Step1: Given Quadratic Equation in the standard form $ax^2 + bx + c = 0$.

Step 2: Divide both sides by a

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Step 3: Transfer the constant on RHS then add square of the half of the coefficient of x i.e. $\left(\frac{b}{2a}\right)^2$ on both sides

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

$$x^2 + 2\left(\frac{b}{2a}\right)x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

Step 4: Now write LHS as perfect square and simplify the RHS.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step 5: Take the square root on both the sides.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Step 6: Now shift all the constant terms to the RHS and we can calculate the value of x as there is no variable at the RHS.

$$x = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} - \frac{b}{2a}$$

3. Quadratic formula method

In this method, we can find the roots by using quadratic formula. The quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b and c are the real numbers and $b^2 - 4ac$ is called discriminant.

To find the roots of the equation, put the value of a, b and c in the quadratic formula.

Nature of Roots

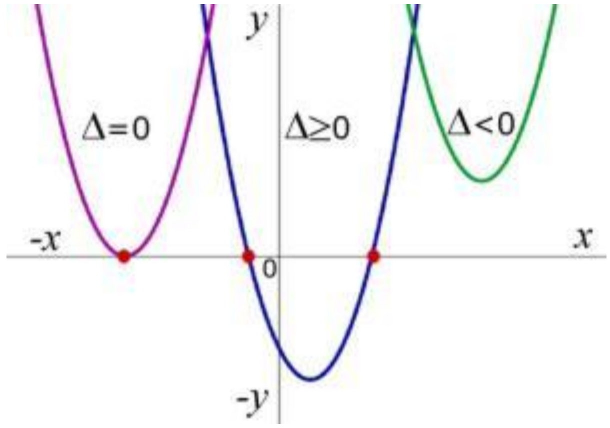
From the quadratic formula, we can see that the two roots of the Quadratic Equation are -

$$x = \frac{-b+\sqrt{b^2-4ac}}{2a} \text{ and } \frac{-b-\sqrt{b^2-4ac}}{2a}$$

$$\text{or } x = \frac{-b\pm\sqrt{D}}{2a}$$

Where **$D = b^2 - 4ac$**

The nature of the roots of the equation depends upon the value of D, so it is called the **discriminant.**



Δ = Discriminant

Value of discriminant	No. of roots	Value of roots
D > 0	Two distinct real roots	$\frac{-b+\sqrt{D}}{2a}, \frac{-b-\sqrt{D}}{2a}$
D = 0	Two equal and real roots	$-\frac{b}{2a}, -\frac{b}{2a}$
D < 0	No real roots	Nil