Revision Notes on Pair of Linear Equations in two variables

Linear Equations

A Linear Equation is an equation of straight line. It is in the form of

$$ax + by + c = 0$$

where a, b and c are the real numbers ($a \neq 0$ and $b \neq 0$) and x and y are the two variables,

Here a and b are the coefficients and c is the constant of the equation.

Pair of Linear Equations

Two Linear Equations having two same variables are known as the pair of Linear Equations in two variables.

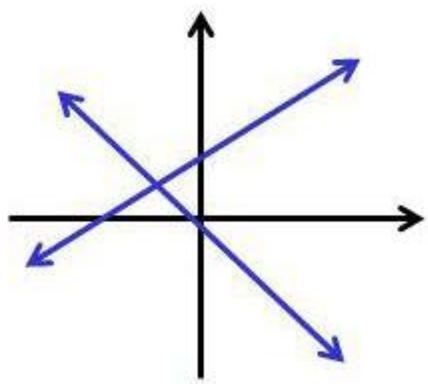
$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

Graphical method of solution of a pair of Linear Equations

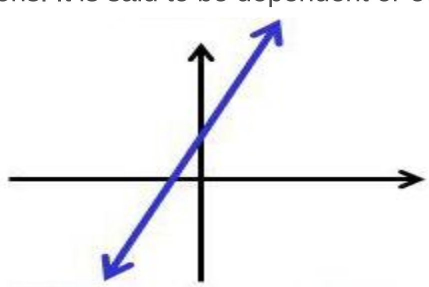
As we are showing two equations, there will be two lines on the graph.

1. If the two lines **intersect** each other at one particular point then that point will be the only solution of that pair of Linear Equations. It is said to be **a consistent** pair of equations.



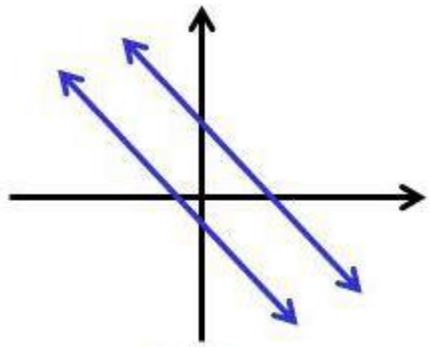
one solution

2. If the two lines **coincide** with each other, then there will be infinite solutions as all the points on the line will be the solution for the pair of Linear Equations. It is said to be dependent or **consistent** pair of equations.



infinitely many solutions

3. If the two lines are **parallel** then there will be no solution as the lines are not intersecting at any point. It is said to be **an inconsistent** pair of equations.



no solution

Algebraic Methods of Solving a Pair of Linear Equations

1. Substitution method

If we have a pair of Linear Equations with two variables x and y, then we have to follow these steps to solve them with the substitution method-

Step 1: We have to choose any one equation and find the value of one variable in terms of other variable i.e. y in terms of x.
Step 2: Then substitute the calculated value of y in terms of x in the other equation.
Step 3: Now solve this Linear Equation in terms of x as it is in one variable only i.e. x.
Step 4: Substitute the calculate value of x in the given equations and find the value of y.

2. Elimination method

In this method, we solve the equations by eliminating any one of the variables.

Step 1: Multiply both the equations by such a number so that the coefficient of any one variable becomes equal.
Step 2: Now add or subtract the equations so that the one variable will get eliminated as the coefficients of one variable are same.
Step 3: Solve the equation in that leftover variable to find its value.
Step 4: Substitute the calculated value of variable in the given equations to find the value of the other variable.

3. Cross multiplication method

Given two equations in the form of

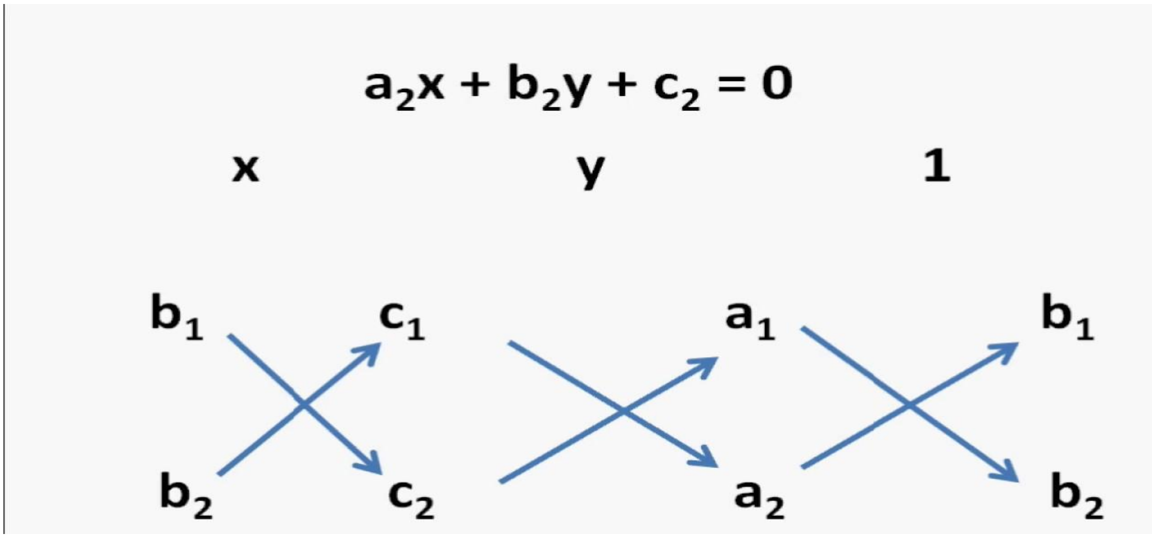
$a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, where

$$x = \frac{(b_1c_2 - b_2c_1)}{(a_1b_2 - a_2b_1)}, y = \frac{(c_1a_2 - c_2a_1)}{(a_1b_2 - a_2b_1)}$$

We write it in general form as

$$\frac{x}{(b_1b_2 - b_2c_1)} = \frac{y}{(c_1a_2 - c_2a_1)} = \frac{1}{(a_1b_2 - a_2b_1)}$$

To apply cross multiplication we use this diagram



The arrows indicate the pairs to be multiplied. The product of the upward arrow pairs is to be subtracted from the product of the downward arrow pairs.

By using this diagram we have to write the equations as given in general form then find the value of x and y by putting the values in the above notations.

y Interpretation of the pair of equations

Pair of Equations	Ratio Comparison	Graphical Representation	Algebraic Interpretation
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Only one solution
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinite solution
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

Equations Reducible to a Pair of Linear Equations in Two Variables

There are some pair of equations which are not linear but can be reduced to the linear form by substitutions.

Given equations

$$\frac{a}{x} + \frac{b}{y} = c$$

We can convert these type of equations in the form of ax + by + c = 0

Let $\frac{1}{x} = m$ and $\frac{1}{y} = n$

Now after substitution the equation will be

am + bn = c

It can be easily solved by any of the method of solving Linear Equations.