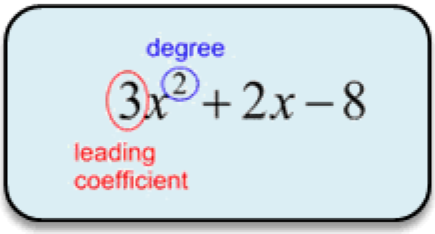
Revision Notes on Polynomials

A polynomial is an expression consists of constants, variables and exponents. It's mathematical form is-

$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + a_2x^2 + a_1x + a_0 = 0$

where the (a_i)'s are constant

Degree of Polynomials



Let P(y) is a polynomial in y, then the highest power of y in the P(y) will be the degree of polynomial P(y).

Types of Polynomial according to their Degrees

Type of polynomial	Degree	Form
Constant	0	P(x) = a
Linear	1	P(x) = ax + b
Quadratic	2	$P(x) = ax^2 + ax + b$
Cubic	3	$P(x) = ax^3 + ax^2 + ax + b$
Bi-quadratic	4	$P(x) = ax^4 + ax^3 + ax^2 + ax + b$

Value of Polynomial

Let p(y) is a polynomial in y and α could be any real number, then the value calculated after putting the value y = α in p(y) is the final value of p(y) at y = α. This shows that p(y) at y = α is represented by p (α).

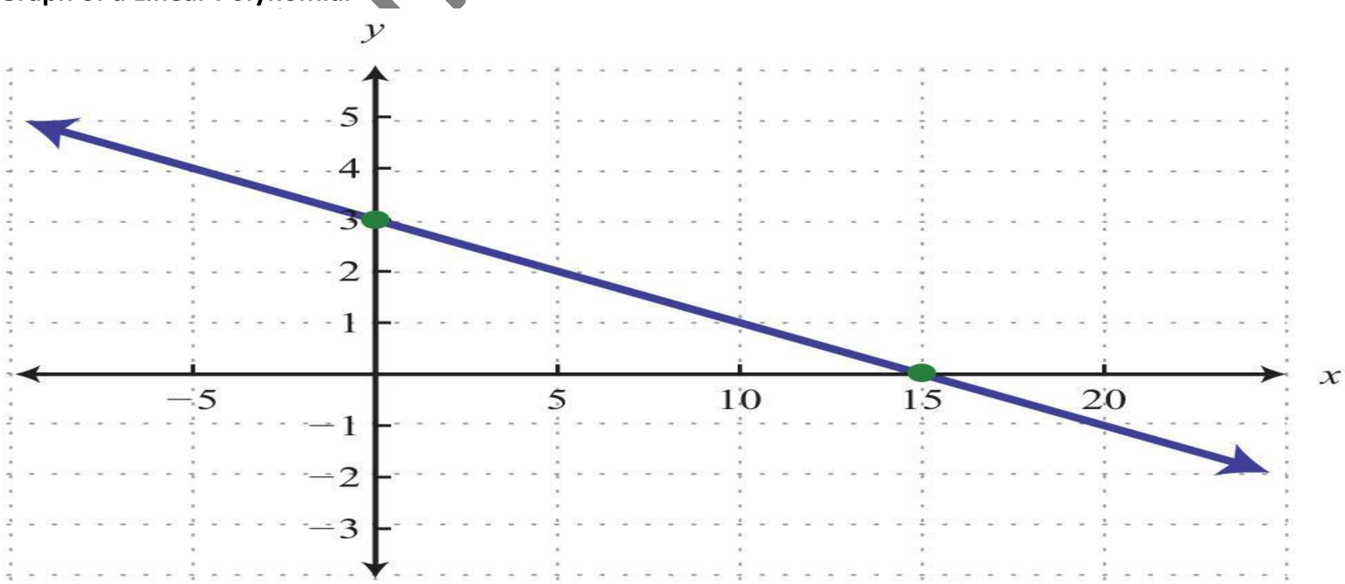
Zero of a Polynomial

If the value of p(y) at y = k is 0, that is p (k) = 0 then y = k will be the zero of that polynomial p(y).

Geometrical meaning of the Zeroes of a Polynomial

Zeroes of the polynomials are the x coordinates of the point where the graph of that polynomial intersects the x-axis.

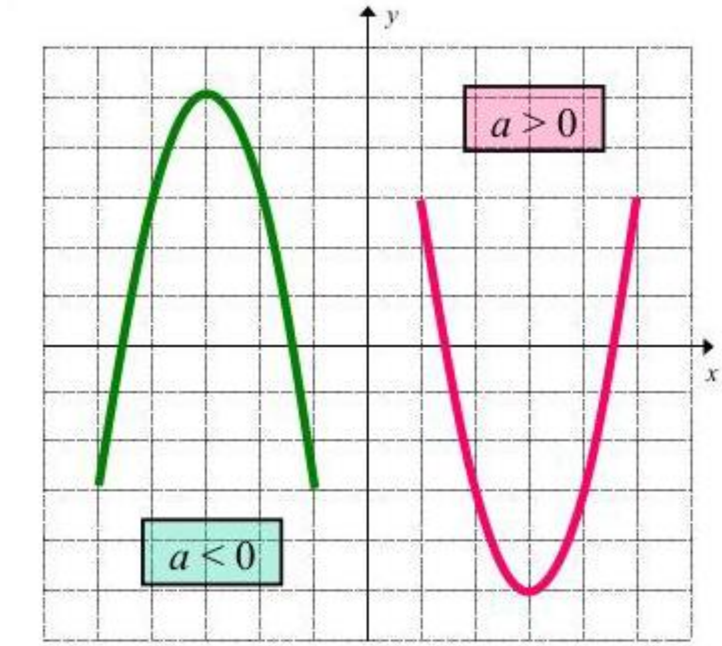
Graph of a Linear Polynomial



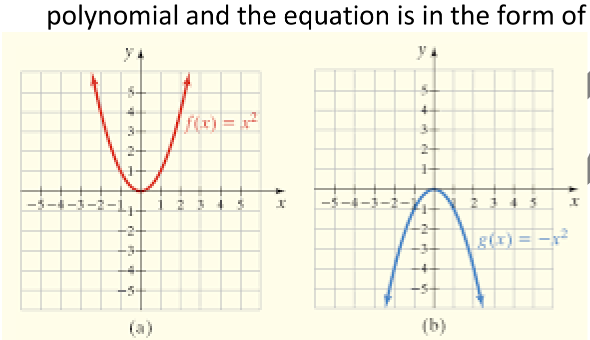
Graph of a linear polynomial is a straight line which intersects the x-axis at one point only, so a linear polynomial has 1 degree.

Graph of Quadratic Polynomial

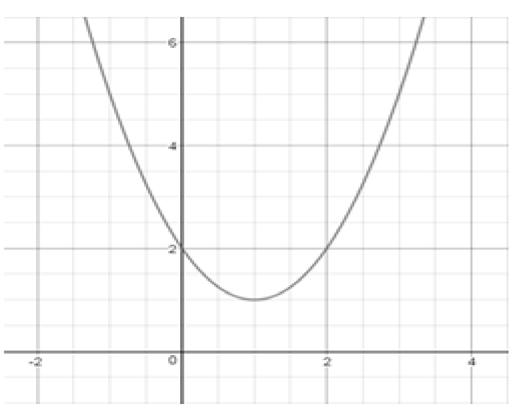
Case 1: When the graph cuts the x-axis at the two points than these two points are the two zeroes of that quadratic polynomial.

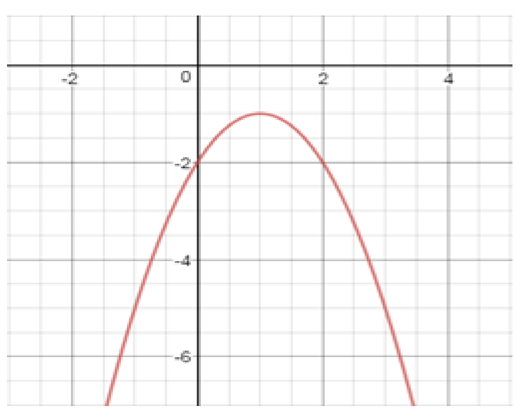


Case 2: When the graph cuts the x-axis at only one point then that particular point is the zero of that quadratic polynomial and the equation is in the form of a perfect square



Case 3: When the graph does not intersect the x-axis at any point i.e. the graph is either completely above the x-axis or below the x-axis then that quadratic polynomial has no zero as it is not intersecting the x-axis at any point.





Hence the quadratic polynomial can have either two zeroes, one zero or no zero. Or you can say that it can have maximum two zero only.

Relationship between Zeroes and Coefficients of a Polynomial

Polynomial	Form	Zeroes	Relationship between Zeroes and Coefficients of a Polynomial
Linear	$ax + b,\ a \neq 0$	1	$k = -\frac{b}{a} = \frac{\text{constant term}}{\text{coefficient of x}}$
Quadratic	$ax^2 + ax + b, a \neq 0$	2	Sum of zeroes $(\alpha + ß) = -\frac{\text{constant of x}}{\text{coefficient of }x^2} = -\frac{b}{a}$ Product of zeroes $(\alpha ß) = \frac{\text{constant term}}{\text{coefficient fo }x^3} = \frac{c}{a}$
Cubic	$ax^3 + ax^2 + ax + b\ ,\ a \neq 0$	3	Sum of zeroes $(\alpha + ß + \gamma) = -\frac{\text{coefficient of }x^2}{\text{coefficient of }x^3} = -\frac{b}{a}$ Product of sum of zeroes taken two at a time $(\alpha ß + ß\gamma + \gamma\alpha) =) = \frac{\text{ccoefficient of x}}{\text{coefficient of }x^3} = \frac{c}{a}$ Product of zeroes$(\alpha ß\gamma) = -\frac{\text{constant term}}{\text{coefficient of }x^3} = -\frac{c}{a}$

Division Algorithm for Polynomial

If p(x) and g(x) are any two polynomials with g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that

P(x) = g(x) × q(x) + r(x),

where r(x) = 0 or degree of r(x) < degree of g(x).