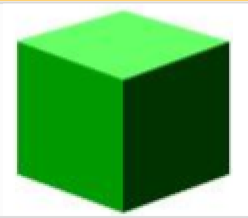
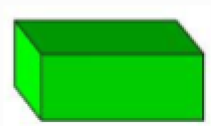
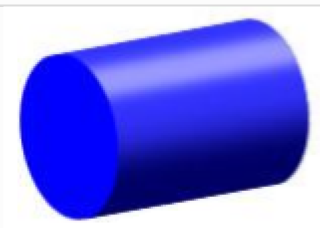
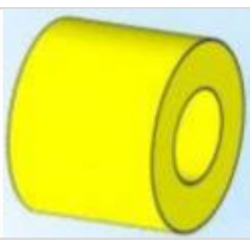
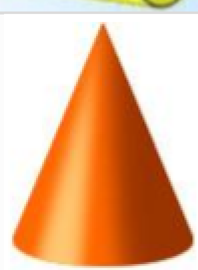
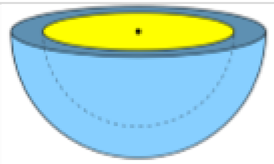
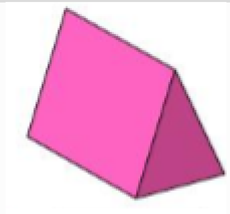
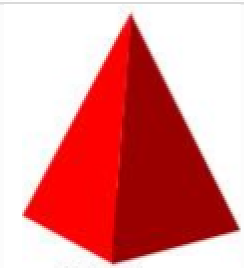
Revision Notes on Surface Areas and Volumes

Surface Areas and Volumes

Surface Area is the area of the outer part of any 3D figure and **Volume** is the capacity of the figure i.e. the space inside the solid. To find the surface areas and volumes of the combination of solids, we must know the surface area and volume of the solids separately. Some of the **formulas** of solids are -

Name	Figure	Lateral or Curved Surface Area	Total Surface Area	Volume	Length of diagonal and nomenclature
Cube		$4l^2$	$6l^2$	l^3	$\sqrt{3}$ l = edge of the cube
Cuboid		2h(l +b)	2(lb + bh + hl)	lbh	$l^2 + b^2 + h^2$ l = length b = breadth h = height
Cylinder		$2\pi rh$	$2\pi r^2 + 2\pi h = 2\pi r(r + h)$	$\pi r^2 h$	r = radius h = height
Hollow cylinder		$2\pi h (R + r)$	$2\pi h (R + r) + 2\pi h (R^2 - r^2)$	-	R = outer radius r = inner radius
Cone		$\pi r l = \pi\sqrt{h^2 + r^2}$	$\pi r^2 + \pi rl = \pi r(r + l)$	$1/3\ \pi r^2 h$	r = radius h = height l = slant height
Sphere		$4\pi r^2$	$4\pi r^2$	$4/3\ \pi r^3$	r = radius
Hemisphere		$2\pi r^2$	$3\pi r^2$	$2/3\ \pi r^3$	r = radius

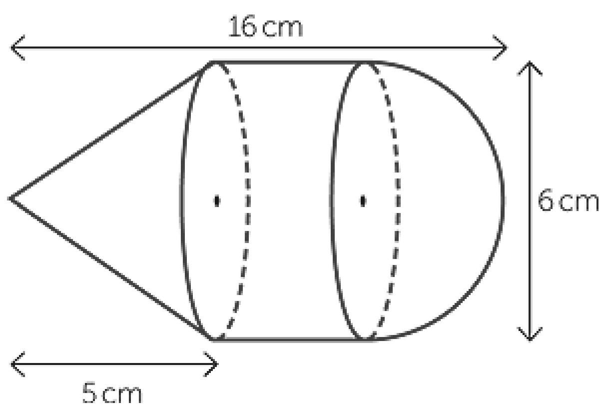
Spherical shell		$4\pi R^2$ (Surface area of outer)	$4\pi r^2$ (Surface area of outer)	$4/3\ \pi(R^3 - r^3)$	R = outer radius r = inner radius
Prism		Perimeter of base × height	Lateteral surface area + 2(Area of the end surface)	Area of base × height	-
pyramid		1/2 (Perimeter of base) × slant height	Lateral surface area + Area of the base	1/3 area of base × height	-

Surface Area of a Combination of Solids

If a solid is molded by two or more than two solids then we need to divide it in separate solids to calculate its surface area.

Example

Find the total surface area of the given figure.



Solution

This solid is the combination of three solids i.e.cone, cylinder and hemisphere.

Total surface area of the solid = Curved surface area of cone + Curved surface area of cylinder + Curved surface area of hemisphere

Curved surface area of cone $= \pi r\sqrt{h^2 + r^2}$

Given, h = 5cm, r = 3cm (half of the diameter of hemisphere)

Curved surface area of cylinder = 2πrh

Given, h = 8cm (Total height – height of cone – height of hemisphere), r = 3cm

Curved surface area of hemisphere = $2\pi r^2$

Given, r = 3 cm

Total surface area of the solid

$$= \left[\pi r\sqrt{h^2 + r^2}\right] + 2\pi rh + 2\pi r^2$$

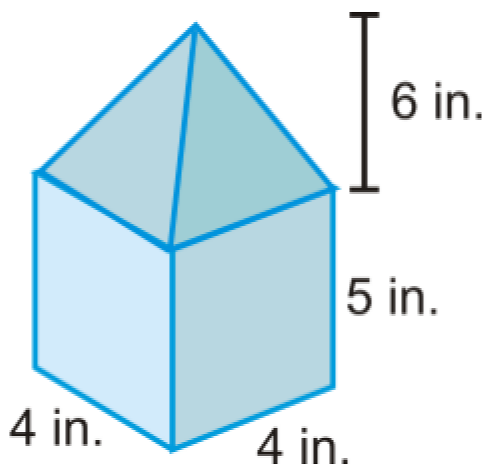
$$= \left[\pi(3)\sqrt{5^2 + 3^2}\right] + 2\pi(3)(8) + 2\pi 3^2$$

$$= 3\sqrt{34}\pi + 48\pi + 18\pi$$

$$= \left(66 + 3 + \sqrt{34}\right)\pi \text{ cm}^2$$

Volume of a combination of solids

Find the volume of the given solid.



Solution

The given solid is made up of two solids i.e. Pyramid and cuboid.

Total volume of the solid = Volume of pyramid + Volume of cuboid

Volume of pyramid = 1/3 Area of base x height

Given, height = 6 in. and length of side = 4 in.

Volume of cuboid = lbh

Given, l = 4 in., b = 4 in, h = 5 in.

Total volume of the solid = 1/3 Area of base x height + lbh

= 1/3 x 4 x 4 x 6 + (4) (4) (5)

= 32 + 80

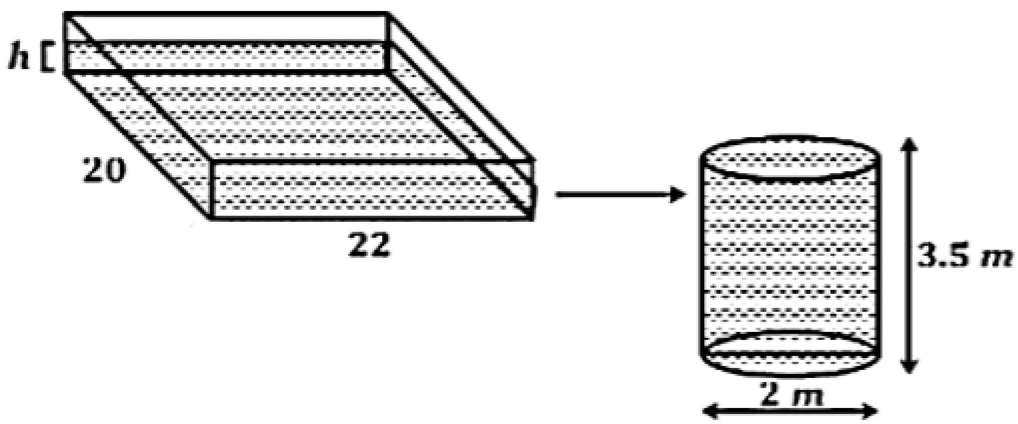
= 112 in^3

Conversion of Solid from One Shape to Another

When we convert a solid of any shape into another shape by melting or remoulding then the volume of the solid remains the same even after the conversion of shape.

Example

If we transfer the water from a cuboid-shaped container of 20 m x 22 m into a cylindrical container having a diameter of 2 m and height of 3.5 m. then what will be the height of the water level in the cuboid container if the cylindrical tank gets filled after transferring the water.



Solution

We know that the volume of the cuboid is equal to the volume of the cylinder.

Volume of cuboid = volume of cylinder

l x b x h = $\pi r^2 h$

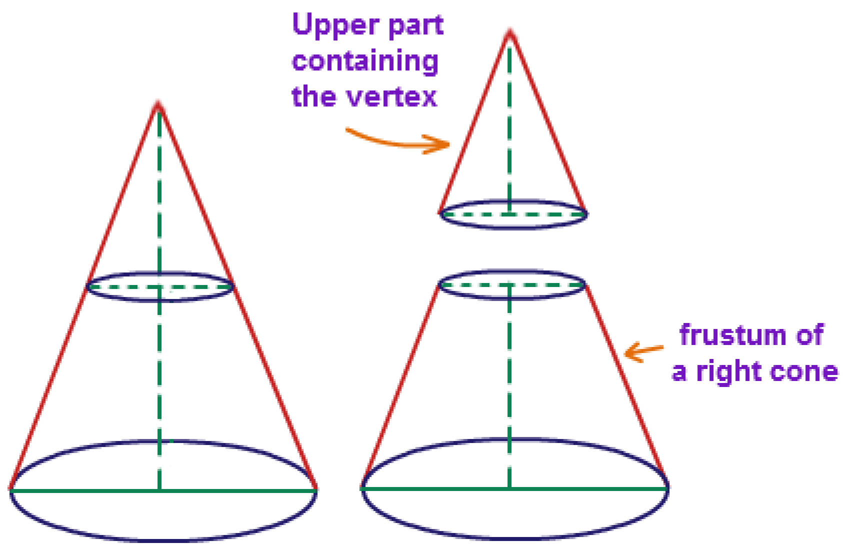
20 x 22 x h = 22/7 x 1 x 3.5

440 × h =11

H = 2.5 cm

Frustum of a Cone

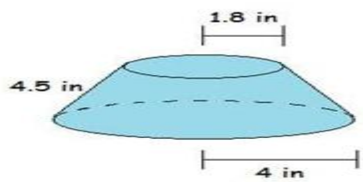
If we cut the cone with a plane which is parallel to its base and remove the cone then the remaining piece will be the Frustum of a Cone.



Volume of the frustum of the cone	$\frac{1}{3}\pi h(R^2 + r^2 + Rr)$
The curved or Lateral surface area of the frustum of the cone	$\pi l(R + r)$
Total surface area of the frustum of the cone	Area of the base + Area of the top + Lateral surface area $\pi R^2 + \pi r^2 + \pi l(R + r)$
Slant height of the frustum	$l = \sqrt{h^2 + (R - r)^2}$

Example

Find the lateral surface area of the given frustum of a right circular cone.



Solution

Given, r =1.8 in.

R = 4 in.

l = 4.5 in.

The lateral surface area of the frustum of the cone = πl (R + r)

= π x 4.5 (4 +1.8)

=3.14 x 4.5 x 5.8

= 81.95 sq. in.