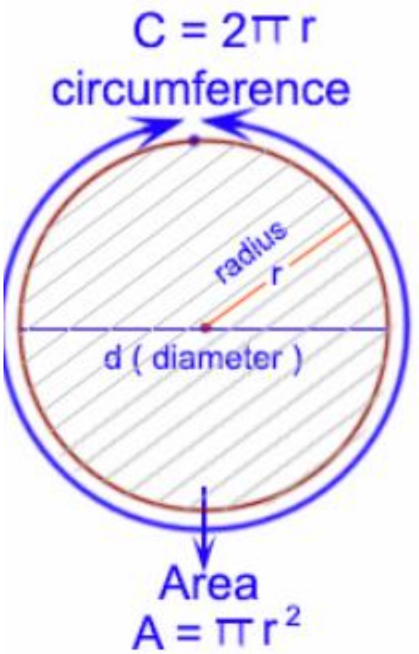
Revision Notes on Areas Related to Circles
Perimeter and Area of a Circle



The perimeter of all the plain figures is the outer boundary of the figure. Likewise, the outer boundary of the circle is the perimeter of the circle. The perimeter of circle is also called the **Circumference of the Circle**.

Circumference = 2πr = πd (π = 22/7)

r = Radius and d = 2r

The area is the region enclosed by the circumference.

Area of the circle = πr^2

Example

If a pizza is cut in such a way that it divides into 8 equal parts as shown in the figure, then what is the area of each piece of the pizza? The radius of the circle shaped pizza is 7 cm.

Solution

The pizza is divided into 8 equal parts, so the area of each piece is equal.

Area of 1 piece = 1/8 of area of circle

$= \frac{1}{8} \times \pi r^2$

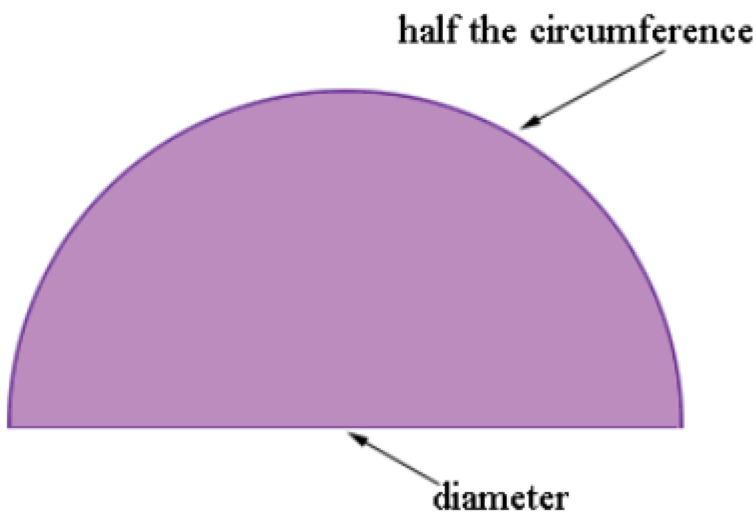
$= \frac{1}{8} \times \frac{22}{7} \times 7 \times 7$

$= \frac{77}{4}\ cn^2$

Perimeter and Area of the Semi-circle

The perimeter of the semi-circle is half of the circumference of the given circle plus the length of diameter as the perimeter is the outer boundary of the figure.

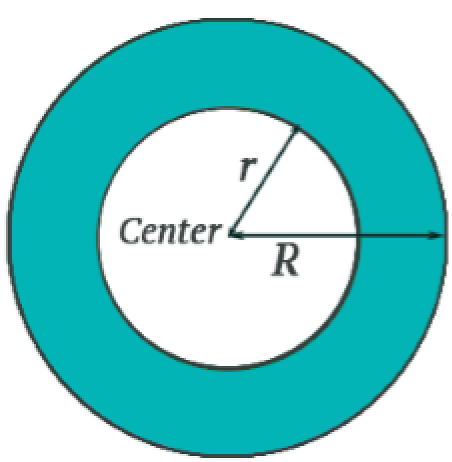
$$\text{Circumference of semi} - \text{circle} = \frac{2\pi}{2} + d = \pi r + 2r$$



Area of the semi-circle is just half of the area of the circle.

$$\text{Area of semi} - \text{circle} = \frac{\pi r^2}{2}$$

Area of a Ring



Area of the ring i.e. the coloured part in the above figure is calculated by subtracting the area of the inner circle from the area of the bigger circle.

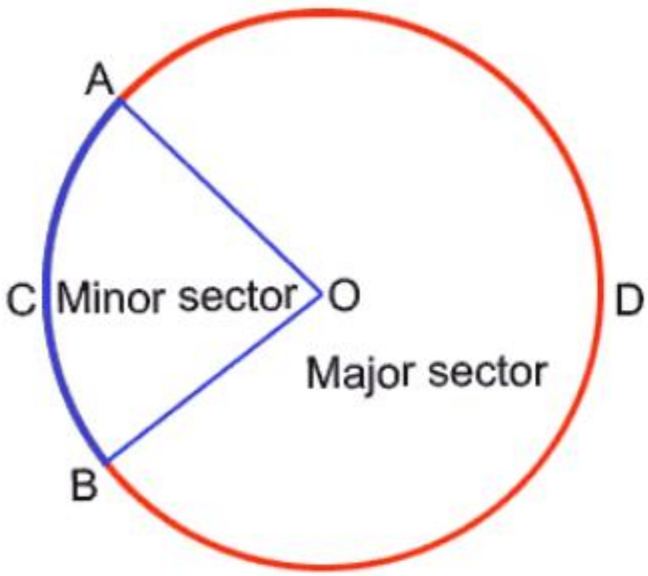
$$\text{Area of the ring} = \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$$

Where, R = radius of outer circle

r = radius of inner circle

Areas of Sectors of a Circle

The area formed by an arc and the two radii joining the endpoints of the arc is called **Sector.**



Minor Sector

The area including ∠AOB with point C is called **Minor Sector.** So OACB is the minor sector. ∠AOB is the angle of the minor sector.

$$\text{Area of the sector of angle } \theta = \frac{\theta}{360} \times \pi r^2$$

Major Sector

The area including ∠AOB with point D is called the **Major Sector**. So OADB is the major sector. The angle of the major sector is 360° – ∠AOB.

Area of Major Sector = πr^2 - Area of the Minor Sector

Remark: Area of Minor Sector + Area of Major Sector = Area of the Circle

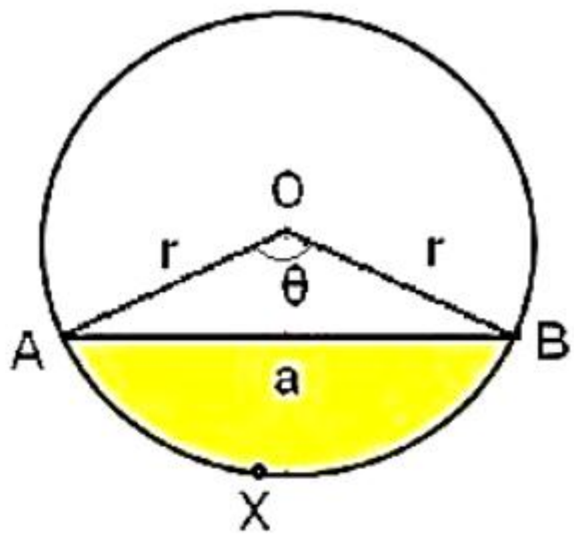
Length of an Arc of a Sector of Angle θ

An arc is the piece of the circumference of the circle so an arc can be calculated as the θ part of the circumference.

$$\text{Length of an arc of a sector of angle } \theta = \frac{\theta}{360^\circ} \times 2\pi r$$

Areas of Segments of the Circle

The area made by an arc and a chord is called the **Segment of the Circle**.



Minor Segment

The area made by chord AB and arc X is the minor segment. The area of the minor segment can be calculated by

Area of Minor Segment = Area of Minor Sector – Area of ΔABO

$$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta$$

Major Segment

The other part of the circle except for the area of the minor segment is called a **Major Segment**.

Area of Major Segment = πr^2 - Area of Minor Segment

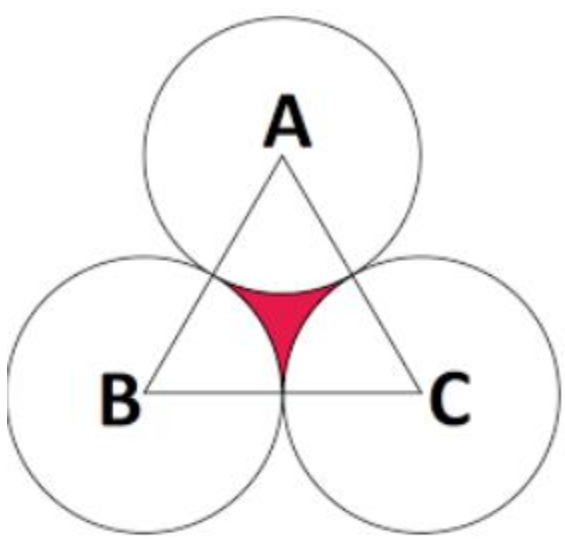
Remark: Area of major segment + Area of minor segment = Area of circle

Areas of Combinations of Plane Figures

As we know how to calculate the area of different shapes, so we can find the area of the figures which are made with the combination of different figures.

Example

Find the area of the colored part if the given triangle is equilateral and its area is 17320.5 cm^2. Three circles are made by taking the vertex of the triangles as the centre of the circle and the radius of the circle is the half of the length of the side of the triangle. (π = 3.14 and √3 = 1.73205)



Solution

Given

ABC is an equilateral triangle, so ∠A, ∠B, ∠C = 60°
Hence the three sectors are equal, of angle 60°.
Required
To find the area of the shaded region.
Area of shaded region =Area of ΔABC – Area of 3 sectors
Area of ΔABC = 17320.5 cm^2

$$\frac{\sqrt{3}}{4} \times (\text{side})^2 = 17320.5$$

$$(\text{side})^2 = \frac{17320.5 \times 4}{1.73205} = 4 \times 10^4$$

Side = 200 cm
As the radius of the circle is half of the length of the triangle, so
Radius = 100 cm

$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

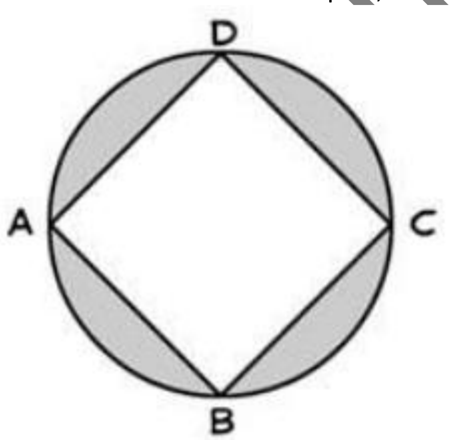
$$= \frac{60°}{360°} \times \pi(100)^2$$

$$= \frac{1}{6} \times 3.14 \times (100)^2 = \frac{15700}{3}\ \text{cm}^2$$

Area of 3 Sectors = 3 × 15700/3 cm^2 cm^2
Area of shaded region = Area of ΔABC – Area of 3 sectors
= 17320.5 - 15700 cm^2
= 1620.5 cm^2

Example

Find the area of the shaded part, if the side of the square is 8 cm and the 44 cm.



Solution

Required region = Area of circle – Area of square
= $\pi r^2 - (\text{side})^2$
Circumference of circle = 2πr = 44

$$2 \times \frac{22}{7} \times r = 44$$

$$r = 44 \times \frac{7}{22} \times \frac{1}{2} = 7$$

Radius of the circle = 7 cm

Area of circle = πr^2

$$\frac{22}{7} \times 7 \times 7 = 154\ \text{cm}^2$$

Area of square = $(\text{side})^2 = (8)^2 = 64\ \text{cm}^2$

Area of shaded region = Area of circle – Area of square

= $154\ \text{cm}^2 - 64\ \text{cm}^2$

= $90\ \text{cm}^2$