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Introduction

In this chapter we will learn about oscillatory motion or oscillations. Any motion which repeats itself at regular intervals of time is known as periodic motion. If a body moves back and forth repeatedly about its mean position then it is said to be in oscillatory motion.
For example: The to and fro movement of pendulum, jumping on a trampoline, a child swinging on a swing.
Oscillations can be defined as Periodic to and fro motion which repeat itself at regular intervals of time.



Pic: To and fro motion of pendulum
Pic: Child on a swing
Pic: Kids jumping on the trampoline

<u>Periodic and Oscillatory motions</u>

Oscillations are defined as to and fro motion which repeat itself after regular intervals of time.In oscillations, the frequency of vibrations iscomparatively less.
For example: The to and fro motion of a pendulum clock

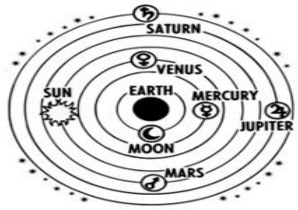

Vibrations are defined as movement of object about its mean position and this motion can be linear, circular, periodic or non-periodic. If vibrations frequency is more and external force also acts on a body.

For eg: - Vibration of guitar string.When we move our fingers on the strings of guitar the strings vibrate so rapidly we can't make it out when it comes to its mean position and when it goes to extreme position because the frequency is very high as a result it has vibratory motion.

Vibrations of the guitar strings

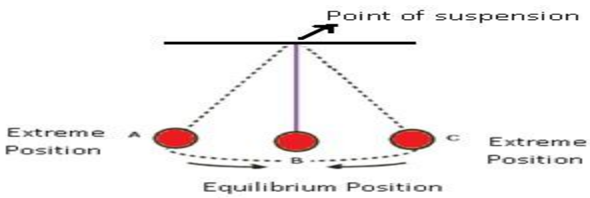
<u>Oscillatory motion and Periodic Motion</u>

Every oscillatory motion is periodic motion that is every oscillatory motion repeats itself after the fixed interval of time. But every periodic motion is not oscillatory.For e.g.:- Motion of planets around the sun is periodic but is not oscillatory motion.



Equilibrium Position

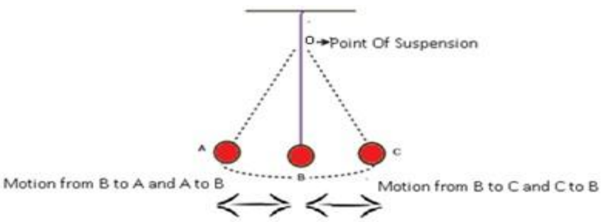
- Oscillating bodies comes to rest at their equilibrium positions. When a bob is suspended from a rigid support it goes to extreme positions and then comes to its mean position which is also known as equilibrium position.
- Equilibrium Position is that position where an object tends to come at rest when no external force is applied.



To and fro motion of the pendulum oscillating from its mean position B to its either extreme positions A and C resp.

Period

- The time taken by an oscillating body to complete one cycle of oscillation.This means the to and fro motion of the body gets repeated after fixed interval of time.
  - It is denoted by T.
  - I. unit is second.



The above imagedescribes the motion of the pendulum, it goes from B to A and then back to B from A. Similarlythe motion of pendulum from B to C.

Frequency

It is defined as number of cycles per second.

- It is denoted by v.
- I.unit is sec^{-1}
- Special Unit is Hertz(Hz)

number of sine waves in one second

Relation between Period and Frequency

$$\nu = 1/T$$

where ν = number of cycles in 1 second

T = 1 cycle

Problem: - On an average human heart is found to be beat 75 times in a minute. Calculate frequency and time period?

Answer: -

The beat frequency of heart = 75/ (1 min)

= 75/ (60 s)

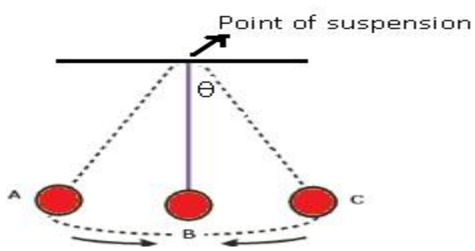
= 1.25 s^{-1}

= 1.25 Hz

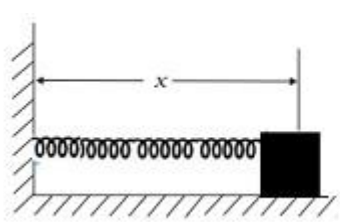
The time period T = 1/ (1.25 s^{-1}) = 0.8 s

Displacement

- Displacement in periodic motion can be represented by a functionwhich is periodic which repeats after fixed interval of time.



In the above image we can see that motion of an oscillating simple pendulum canbe described in terms of angular displacement θ from the vertical.



In the above image we can see that there is a block whose one end is attached to a spring and another is attached to a rigid wall.x is the displacement from the wall.

In the above figure a blockis attached to a spring, the other end of which is fixed to a rigid wall. The block moves on a frictionless surface. The motion of the block can be described in terms of its distance or displacement x from the wall.

f (t) = A cos ωt

As cosine function repeats after 2π so it can be written as

cos (θ) = cos (ωt + 2π) Equation (1)

cos (ωt) = cos (ωt + 2π) (it keep on repeating after 2π)

Let Time Period = T

f (T) = f(t+T) where displacement keeps on repeating after (t+T)

Acos (ωt) = cosω(t+T) = Acos (ωt+ wT)

Acosωt = A cos (ωt+ωT) Equation (2)

From Equation (1) and Equation (2)

ωT= 2π

Or **T=2π /ω**

Displacement as a combination of sine and cosine functions

f (t) = A cos ωt

f (t) = A sin ωt

f (t) = A sin ωt + A cos ωt

LetA = D cosΦ Equation (3)

B=DsinΦEquation (4)

f (t) =DcosΦ sinωT + DsinΦ cos ωt

D (cosΦ sinωT + sinΦ cos ωt)

(Using sinAcosB + sinBcosA = sin (A+B))

Therefore we can write

f (T)= **D sin (ωT+Φ)**

From the above expression we can say displacement can be written as sine and cosine functions.

D in terms of A and B:-

$A^2 B^2 = D^2 sin^2 Φ + D^2 cos^2 Φ$

$A^2 B^2 = D^2$

Or D= AB

Φ In terms of A and B

Dividing Equation (4) by (3)

B/A= DsinΦ/Dcos Φ

tan Φ = B/A

Or Φ= tan^{-1} B/A

Problem:-Which of the followingfunctions of time represent (a) periodic and (b) non-periodic motion? Give the period foreach case of periodic motion [ω is anypositive constant].

(i) sin ωt + cos ωt

(ii) sin ωt + cos 2 ωt + sin 4 ωt

(iii) e–ωt

(iv) log (ωt)

Answer:-

- sin ωt + cos ωt is a periodic function, it can also be written as

2 sin (ωt + π/4).

Now 2 sin (ωt + π/4)= 2 sin (ωt + π/4+2π)

= 2 sin [ω (t + 2π/ω) + π/4]

The periodic time of the function is 2π/ω.

(ii) This is an example of a periodic motion. Itcan be noted that each term represents aperiodic function with a different angularfrequency. Since period is the least intervalof time after which a function repeats itsvalue, sin ωt has a period T_0= 2π/ω; cos 2 ωt

has a period π/ω =T_0/2; and sin 4 ωt has aperiod 2π/4ω = T_0/4. The period of the firstterm is a multiple of the periods of the last

two terms. Therefore, the smallestintervalof time after which the sum of the threeterms repeats is T_0, and thus the sum is aperiodic function with aperiod 2π/ω.

(iii) The function $e^{-\omega t}$ is not periodic, itdecreases monotonically with increasingtime and tends to zero as t →∞ and thus,never repeats its value.
(iv) The function log (ωt) increases monotonicallywith time t. It, therefore, neverrepeats its value and is a non-periodicfunction. It may be noted that as t →∞,log (ωt) diverges to ∞. It, therefore, cannot represent any kind of displacement.

SIMPLE HARMONIC MOTION

Simple Harmonic Motion (SHM) is a periodic motion the body moves to and fro about its mean position.The restoring force on the oscillating body is directly proportional to its displacement and is always directed towards its mean position.

In the above image we can see that a particle is vibrating to and fro within the limits –A and +A.
The oscillatory motion is said to be SHM if the displacement x of the particle from origin varies with time t:

x (t) = A cos (ωt + Φ)

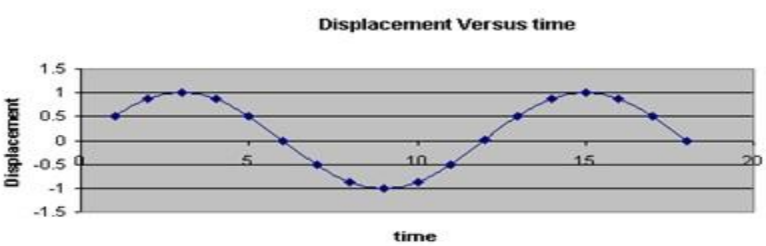
Where
x (t) : displacement x as a function of time
A= amplitude

- It is defined as magnitude of maximum displacement of the particle from its mean position.

ωt +Φ = phase angle (time-dependent)
ω= angular frequency
Φ = phase constant

- SHM is a periodic motion in which displacement is a sinusoidal function of time.

If we plot the graph between displacement versus time we can conclude that the displacement is continuous function of time.



The above graph shows displacement as a continuous function of time.

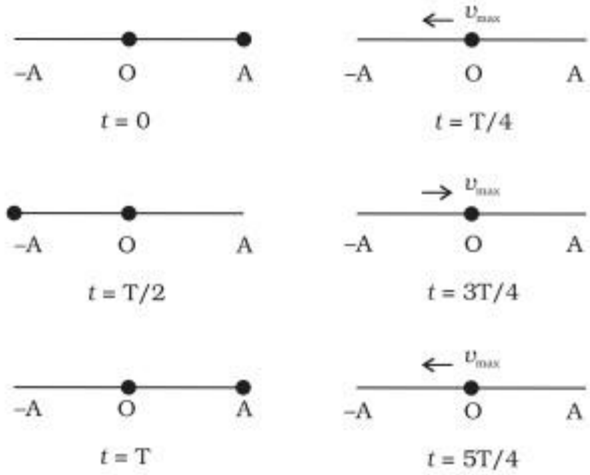
Phase

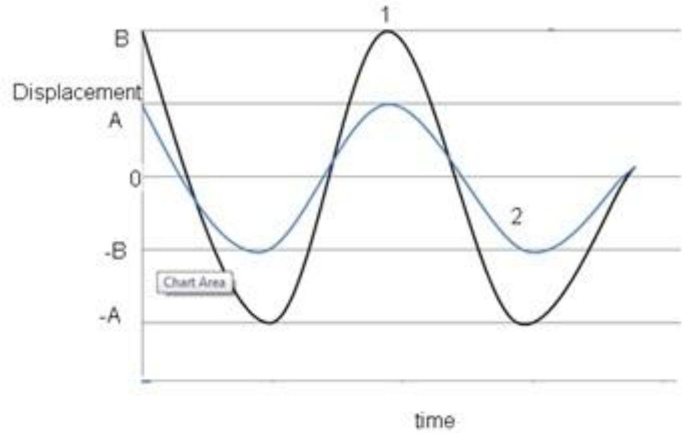
It is that quantity that determines the state of motion of the particle.

- Its value is (ωt + Φ)
- It is dependent on time.

Value of phase at time t=0, is termed as **Phase Constant**. When the motion of the particle starts it goes to one of the extreme position at that time phase is considered as 0.
Let x (t) = A cos (ωt) where we are taking (Φ = 0)

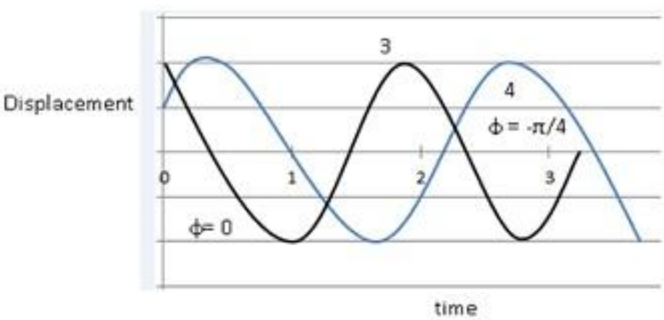
1. **Mean Position** (t= 0)
2. x (0) = A cos (0) = A (cos0=1)
3. t=T/4, t= T/2, t=3T/4, t=T and t=5T/4





The above figures depict the location of the particle in SHM at different values of t=0,T/4,T/2,3T/4,T,5T/4.The time after which motion repeats is T.The speed is maximum for zero displacement(x=0) and zero at the extremes of motion.

In the above graph the displacement as a function of time is obtained when φ = 0.The curves (1) and (2) are of two different amplitudes A and B.



In the above graph the curves (3) and (4) are for φ = 0 and -π/4 respectively but the amplitude is same for both.

Problem: - Which of the following relationships between the acceleration (a) and the displacement x of a particle involve simple harmonic motion?
(a) a = 0.7x
(b) a = –200x2
(c) a = –10x
(d) a = 100x3
Answer:
In SHM, acceleration a is related to displacement by the relation of the form
a = -kx, which is for relation (c).
Problem: - The motion of a particle executing simple harmonic motion is described by the displacement function,
x (t) = A cos (ωt + φ).
If the initial (t = 0) position of the particle is 1 cm and its initial velocity is ω cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is π s^{-1}. If instead of the cosine function, we choose the sine function to describe the SHM: x = B sin (ωt + α), what are the amplitude and initial phase of the particle with the above initial conditions.
Answer:-
Initially, at t = 0;

Displacement, x = 1 cm
Initial velocity, v = ω cm/ sec.
Angular frequency, ω = π rad/s^{-1}
It is given that,
x (t) = A cos (ωt + Φ)
1 = A cos (ω × 0 + Φ) = A cos Φ
A cosΦ = 1 ... (i)
Velocity, v= dx/dt
ω = -A ω sin (ωt + Φ)
1 = -A sin (ω × 0 + Φ) = -A sin Φ
A sin Φ = -1 ... (ii)
Squaring and adding equations (i) and (ii), we get:
A^2 (sin^2 Φ + cos^2 Φ) = 1 + 1
A^2 = 2
∴ A = √2 cm
Dividing equation (ii) by equation (i), we get:
tan Φ = -1
∴Φ = 3π/4, 7π/4...
SHM is given as:
x = B sin (ωt + α)
Putting the given values in this equation, we get:
1 = B sin [ω × 0 + α] = 1 + 1
B sin α = 1 ... (iii)
Velocity, v = ω B cos (ωt + α)
Substituting the given values, we get:
π = π B sin α
B sin α = 1 ... (iv)
Squaring and adding equations (iii) and (iv), we get:
B2 [sin2 α + cos2 α] = 1 + 1
B2 = 2
∴ B = √2 cm
Dividing equation (iii) by equation (iv), we get:
B sin α / B cos α = 1/1
tan α = 1 = tan π/4
∴α = π/4, 5π/4...

Angular Frequency (ω)

Angular frequency refers to the angular displacement per unit time. It can also be defined as the rate of change of the phase of a sinusoidal waveform (e.g., in oscillations and waves). Angular frequency is larger than frequency v (in cycles per second, also called Hz), by a factor of 2π.

Consider the oscillatory motion which is varying with time t and displacement x of the particle from the origin:
x (t) = cos (ωt + Φ)
Let Φ = 0
x (t) = cos (ωt)
After t=T i.e. x (t) = x (t+T)
A cos ωt = A cos ω (t + T)
Now the cosine function is periodic with period 2π, i.e., it first repeats itself after 2π. Therefore,
ω (t + T) = ωt + 2π
i.e. **ω = 2π/ T**
Where ω = angular frequency of SHM.

- Its unit is radians per second.
- It is 2π times the frequency of oscillation.
- Two simple harmonic motions may have the same A and φ, but different ω.

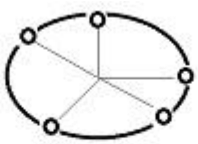
Problem: - Which of the following functions of time represent (a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic motion (ω is any positive constant):
(a) sin ωt – cos ωt
(b) sin3ωt
(c) 3 cos (π/4 – 2ωt)
(d) cos ωt + cos 3ωt + cos 5ωt
(e) exp ($-\omega^2 t^2$)
Answer: -
(a) SHM
The given function is:
sinωt – cos ωt
This function represents SHM as it can be written in the form: a sin (ωt + Φ)
Its period is: 2π/ω
(b) Periodic but not SHM
The given function is:
sin 3ωt = 1/4 [3 sin ωt - sin3ωt]
The terms sin ωt and sin ωt individually represent simple harmonic motion (SHM). However, the superposition of two SHM is periodic and not simple harmonic.
Its period is: 2π/ω
(c) SHM
The given function is:
This function represents simple harmonic motion because it can be written in the form: a cos (ωt + Φ) its period is: 2π/2ω = π/ω
(d) Periodic, but not SHM
The given function is cosωt + cos3ωt + cos5ωt. Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic.
(e) Non-periodic motion
The given function exp ($-\omega^2 t^2$) is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion.
(f) The given function $1 + \omega t + \omega^2 t^2$ is non-periodic.

Problem: Which of the followingfunctions of time represent (a) simple harmonic motion and (b) periodic but notsimple harmonic? Give the period for each case?
(1) sin ωt – cos ωt
(2) $\sin^2 \omega t$
Answer:
(a) sin ωt – cos ωt
= sin ωt – sin (π/2 – ωt)
= 2 cos (π/4) sin (ωt – π/4)
= √2 sin (ωt – π/4)
This function represents a simple harmonicmotion having a period T = 2π/ω and aphase angle (–π/4) or (7π/4).
(b) $\sin^2 \omega t$
= ½ – ½ cos 2 ωt
The function is periodic having a periodT = π/ω. It also represents a harmonicmotion with the point of equilibriumoccurring at ½ instead of zero.

SHM & Uniform Circular Motion

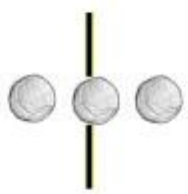
Uniform Circular motion can be interpreted as a SHM.

To explain above statement:- Consider a ball tied to a thread and moving in a circular path such a way it appears to be in circular motion for a person who is observing it from top view or who is standing in the same plane as we are. But it appears to be SHM if somebody is standing in the same plane of motion.



Case 1: It is executing circular motion.

Case 2: Circular motion observed by a person when standing on the same plane.



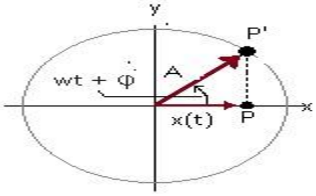
Case 3: It is appears to be SHM for a person who is standing in the same line of sight.
Mathematically:-
Consider any particle moving in a circular path whose radius is R
Angular velocity = w
Angular position =∫θ dt
=wt + φ
Consider the projection of particle on x-axis be P'.
Displacement = Acosθ

x=A cos(wt+ φ)

the above equation same as the equation of simple harmonic motion

As the particle is moving in the same way the projections are also moving.

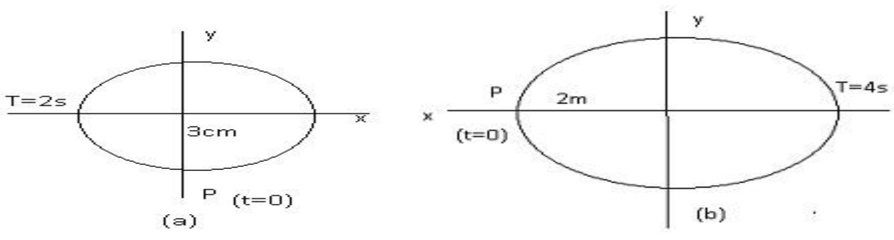
- When the particle is moving in the upper part of circle then the projections start moving towards left.
- When the particle is moving in the lower part of the circle then the projections are moving towards right.
- We can conclude that the particle is swinging from left to right and again from right to left.
- This to and fro motion is SHM.



In the above figure we can see that a reference point P ' moving with uniform circular motion in a reference circle of radius A. Its projection P on the x-axis executes simple harmonic motion.

Conclusion: -
SHM is the projection of uniform circular motion on the diameter of the circle in which the SHM takes place.

Problem:-In the given figuresit corresponds to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e. clockwise or anti-clockwise) are indicated on each figure?



Obtain the corresponding simple harmonic motions of the x-projection of the radius vector of the revolving particle P, in each case.

Answer: - (a) Time period, t = 2 s
Amplitude, A = 3 cm
At time, t = 0, the radius vector OP makes an angle π/2 with the positive x-axis, phase angle Φ = +π/2
Therefore, the equation of simple harmonic motion for the x-projection of OP, at the time t, is given by the displacement equation:

$A= \cos [(2 \pi t/T) + \Phi]$

$=3\cos (2 \pi t/2 + \pi/2) = -3\sin (2\pi t/2)$

$=-3\sin\pi t$ cm.

(b) Time Period, t = 4 s
Amplitude, a = 2 m
At time t = 0, OP makes an angle π with the x-axis, in the anticlockwise direction, Hence, phase angle Φ = +π
Therefore, the equation of simple harmonic motion for the x-projection of OP, at the time t, is given as**:**

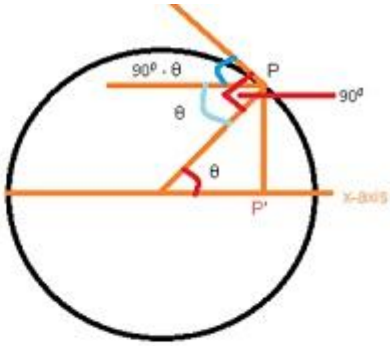
$=a \cos [(2 \pi t/T) + \Phi]$

$=2 \cos [(2 \pi t/T) + \pi]$

$x=-2 \cos (\pi/2\ t)$ m

Velocity in Simple Harmonic Motion

- Uniform Circular motion can be defined as motion of an object in a circle at a constant speed.
- Consider a particle moving in circular path
- The velocity at any point P at any time t will be tangential to the point P.
- Consider $\theta = \omega t+ \varphi$ where
- θ = angular position
- ω = angular velocity of the particle



- To calculate the value of velocity along the x-axis for theprojectionP'which is executing SHM.
- To find the component of velocity along -ive x-axis, we can write = -v cos(90^0 - θ)

This is can be written as:

- = -v cos [90^0 – (ωt + φ)]
- **v (t)** = **- v sin(ωt**+ **φ)** (Equation 1)

where

- v (t) is instantaneous velocity of the particle executing the SHM.
- (-ive sign tells that the velocity is directed towards negative x-axis)
- To verify whether the Equation(1)is same if we calculate directly from SHM:-

x(t) = A cos(ωt + φ)

where

- x(t) = displacement vector

v (t) = dx/dt

where

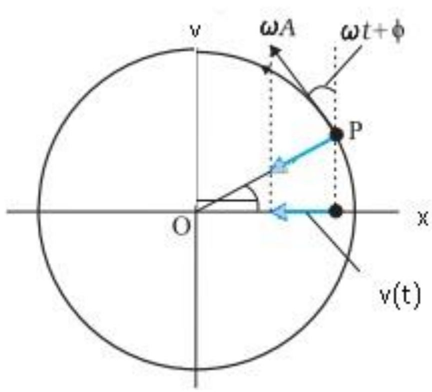
- v(t) = velocity
- dx/dt is rate of change of displacement

= -Awsin (wt + φ)

v (t) = **- A w sin(wt** + **φ)** (Same as Equation(1))

- We can see from the above equation that the radius of the circular motionbecomes the amplitude of the SHM.

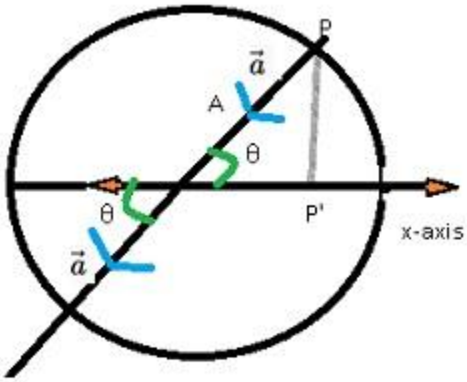
Instantaneous velocity of the particle executing SHM is given as:- v (t) = - A w sin(wt + φ)



In the above figure we can see that the velocity, v (t), of the particle P' is the projection of the velocity v of the reference particle, P.

Acceleration in Simple Harmonic Motion

- Acceleration in uniform circular motion always directed towards the centre. It is known as radial acceleration.
- $a_p = -\omega^2 r$ where r = radius



- Consider a particle moving in a circular path. Particle is at some point P at some instant of time, radius of circular path is equal to amplitude.
- Acceleration will be given as

$a_p = -\omega^2 A$where

- A = radius of the circle
- (-ive sign shows it is pointing towards the centre of the circle.)
- Consider the acceleration of the projection of the particle P' on the x-axis.
- Accelerationwill be given as
- $a(t) = -a_p \cos\theta$
- $a(t) = -\, a_p \cos(\omega t + \varphi)$
- $=\omega^2 A \cos(\omega t + \varphi)$
- $\mathbf{a(t) = -\, \omega^2 x(t)}$ (Using $x(t)= A \cos(\omega t + \varphi)$)

To verify expression for acceleration when calculated directly from SHM -

- Displacement in SHM is = $A \cos(\omega t + \varphi)$
- Velocity v(t) in SHM is = $-A w \sin(\omega t + \varphi)$

Therefore,

- $a(t) = dv/dt$
- $= -\, \omega^2 A \cos(\omega t + \varphi)$
- $a(t) = -\, \omega^2 x(t)$ (Using $x(t)= A \cos(\omega t + \varphi)$)

Equation of acceleration of the particle which executesSHM:-

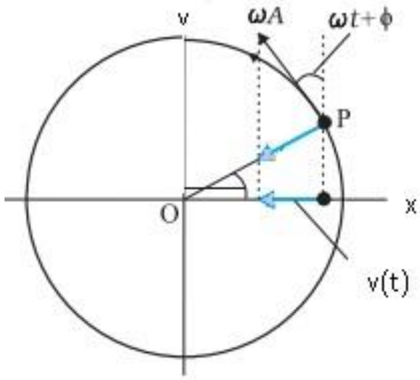
$\mathbf{a(t) = -\, \omega^2 x(t)}$

We can conclude that:-

1. a is proportional to displacement
2. acceleration is always directed towards the centre(in circular motion centre is mean position of the SHM)

From above we can say that

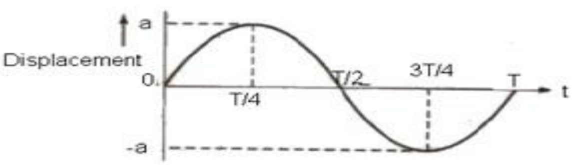
- SHM is the projection of the uniform circular motion such that centre of uniform circular motion becomes the mean position of the SHM and the radius of the circular motion is the amplitude of the SHM.

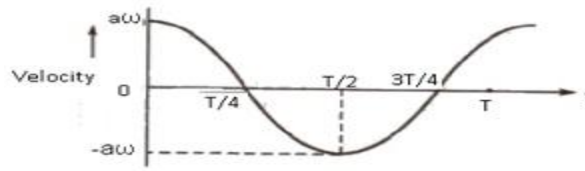


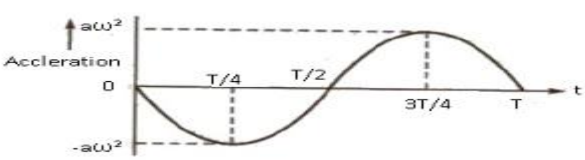
In the above figure we can see that the acceleration, a(t), of the particle P′ is the projection of the acceleration a of the reference particle P.

In all the below graphs displacement, velocity and acceleration all have the same time period T, but they differ in phase.

The acceleration is maximum where velocity is minimum and vice-versa.







Problem: -A body oscillates with SHM according to the equation (in SI units), $x = 5 \cos [2\pi t + \pi/4]$
At t = 1.5 s, calculate the (a) displacement,(b) speed and (c) acceleration of the body?
Answer:
The angular frequency ω of the body
$= 2\pi\ s^{-1}$
and its time period T = 1 s.
At t = 1.5 s
(a) Displacement $= (5.0\ m) \cos [(2\pi\ s^{-1})1.5\ s + \pi/4]$
$= (5.0\ m) \cos [(3\pi + \pi/4)]$
$= -5.0 \times 0.707\ m$
$= -3.535\ m$

(b) The speed of the body
$= -(5.0\ m)(2\pi\ s^{-1}) \sin [(2\pi\ s^{-1})\ 1.5\ s + \pi/4]$

$= -(5.0\ m)(2\pi\ s{-}1)\sin[(3\pi + \pi/4)]$
$= 10\pi\ (0.707)\ m\ s^{-1}$
$= 22\ m\ s^{-1}$

(c) The acceleration of thebody
$= -(2\pi\ s^{-1})^2$displacement
$= -(2\pi\ s^{-1})^2\ (-3.535\ m)$
$= 140\ m\ s^{-2}$

Problem: -Two identical springs of spring constant k are attached to a block of mass m and to fixed supports as shown in Fig. Show that when the mass is displaced from its equilibrium position on either side, it executes a simple harmonic motion. Find the period of oscillations?

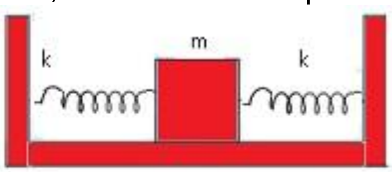
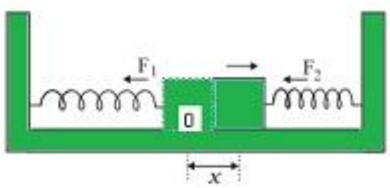


Fig (a)
Answer:-



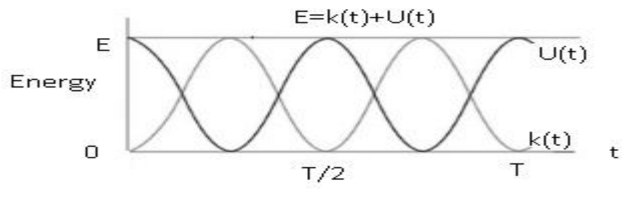
Let the mass be displaced by a smalldistance x to the right side of the equilibriumposition, as shown in Fig (a). Under thissituation the spring on the left side getselongated by a length equal to x and that onthe right side gets compressed by the samelength. The forces acting on the mass arethen,
$F_1 = -kx$ (force exerted by the spring onthe left side, trying to pull the mass towards the mean position)
$F_2 = -kx$ (force exerted by the spring onthe right side, trying to pull the mass towards the mean position)
The net force, F, acting on the mass is thengiven by,
$F = -2kx$
Therefore, the force acting on the mass is proportional to the displacement and is directedtowards the mean position; therefore, the motion executed by the mass is simple harmonic. The time period of oscillations is given as:-
$T=2\pi\sqrt{m/2k}$

Problem:-The acceleration due to gravity on the surface of moon is $1.7\ ms^{-2}$. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is $9.8\ ms^{-2}$)
Answer: -Acceleration due to gravity on the surface of moon,$g' = 1.7\ m\ s^{-2}$
Acceleration due to gravity on the surface of earth, $g = 9.8\ m\ s^{-2}$
Time period of a simple pendulum on earth, T = 3.5 s
$T=2\pi\sqrt{l/g}$
where l =length of the pendulum
$l= T^2/(2\pi)^2 x\ g$
$= (3.5)^2/(4x\ (3.14)^2)\ x\ 9.8\ m$
The length of the pendulum remains constant,
On moon's surface, time period, $T' = 2\pi\sqrt{l/g'}$
$=2\pi\ ((3.5)^2/4x(3.14)^2\ x\ 9.8)/1.7$

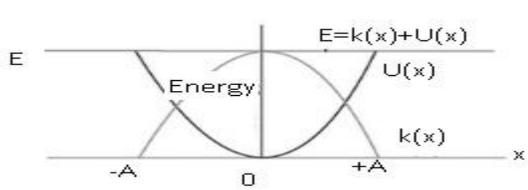
=8.4s
Hence, the time period of the simple pendulum on the surface of moon is 8.4 s.

Problem:-
(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle: $T = 2\pi\sqrt{(m/k)}$. A simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?
(b) The motion of a simple pendulum is approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than $2\pi\sqrt{(l/g)}$
Think of a qualitative argument to appreciate this result.
(c) A man with a wristwatch on his hand falls from the top of a tower. Does the watch give correct time during the free fall?
(d) What is the frequency of oscillation of a simple pendulum mounted in a cabinthat is freely falling under gravity?
Answer:
(a) For a simple pendulum, force constant or spring factor k is proportional to mass m; therefore, m cancels out in denominator as well as in numerator. That is why the time period of simple pendulum is independent of the mass of the bob.
(b) In the case of a simple pendulum, the restoring force acting on the bob of the pendulum is given as:
$F = -mg \sin\theta$
where,
F = Restoring force
m = Mass of the bob
g = Acceleration due to gravity
θ = Angle of displacement
For small θ, $\sin\theta \approx \theta$
For large θ, $\sin\theta$ is greater than θ.
This decreases the effective value of g.
Hence, the time period increases as:
$T = 2\pi\sqrt{(l/g)}$
(c) Yes, because the working of the wrist watch depends on spring action and it has nothing to do with gravity.
(d) Gravity disappears for a man under free fall, so frequency is zero

Energy in SHM
The Kinetic and Potential energies in a SHM varies between 0 and their maximum values.



Kinetic energy, potential energy and the total energy is a function of time in the above graph.
BothKinetic energy and potential energy repeats after time T/2.



Kinetic energy, potential energy and the total energy is a function of displacement in the above graph.

The kinetic energy (K) of a particle executing SHM can be defined as

$K = ½ mv^2$
$= ½ m\omega^2 A^2 \sin^2 (\omega t + \varphi)$
$K = ½ k A^2 \sin^2 (\omega t + \varphi)$

- The above expression is a periodic function of time, being zero when the displacement is maximum and maximum when the particle is at the mean position.

The potential energy (U) of a particleexecuting simple harmonic motion is,

$U(x) = ½ kx^2$
$U = ½ k A^2 \cos^2 (\omega t + \varphi)$

- The potential energy of a particle executing simple harmonic motion is alsoperiodic, with period T/2, being zero at the mean position and maximum at the extremedisplacements.

Total energy of the system always remains the same

$E = U + K$

$= ½ k A^2 \sin^2 (\omega t + \varphi) + ½ k A^2 \cos^2 (\omega t + \varphi)$
$E = ½ k A^2 (\sin^2 (\omega t + \varphi) + \cos^2 (\omega t + \varphi))$

The above expression can be written as

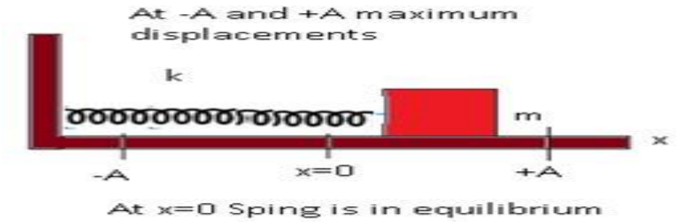
$E = ½ k A^2$
Total energy is always constant.

Problem: -A block whose mass is 1 kgis fastened to a spring. The spring has a spring constant of 50 N m^{-1}. The block ispulled to a distance x = 10 cm from its equilibrium position at x = 0 on a frictionlesssurface from rest at t = 0. Calculate thekinetic, potential and total energies of theblock when it is 5 cm away from mean position?
Answer: -The block executes SHM, its angularfrequency, according to equation, $\omega = \sqrt{k/m}$
$= \sqrt{(50 \text{ N m}^{-1})/ 1\text{kg}}$
$= 7.07 \text{ rad s}^{-1}$
Its displacement at any time t is then given by,
x (t) = 0.1 cos (7.07t)
Therefore, when the particle is 5 cm away fromthe mean position, we have
0.05 = 0.1 cos (7.07t)
Or cos (7.07t) = 0.5 and hence
sin (7.07t) = $\sqrt{3}/2$= 0.866
Then, the velocity of the block at x = 5 cm is
$= 0.1\ 7.07\ 0.866 \text{ m s}^{-1}$
$= 0.61 \text{ m s}^{-1}$
Hence the K.E. of the block,
$=1/2\ m v^2$
$= [1\text{kg} (0.6123 \text{ m s}^{-1})^2]$
= 0.19 J
The P.E. of the block,
$=1/2\ kx^2$
$= (50 \text{ N m}^{-1}\ 0.05 \text{ m}\ 0.05 \text{ m})$
= 0.0625 J

The total energy of the block at x = 5 cm,

= K.E. + P.E.
= 0.25 J
we also know thatmaximum displacement,K.E. is zero and hence the total energy of thesystem is equal to the P.E. Therefore, the totalenergy of the system,
= (50 N m^{-1} 0.1 m 0.1 m)
= 0.25 J , which is same as the sum of the two energies ata displacement of 5 cm.This shows the result with the accordance of conservation of energy.

Oscillations due to spring
Consider a block if it is pulled on one side and is released, and then it executes to and fro motionabout a mean position.



In the above image ablock, is on a frictionless surface when pulled or pushed and released, executes simple harmonic motion.
F (x) = –k x (expression for restoring force)

- kis known as **spring constant**and its value is governed by the elastic properties of the spring.
- The above expression is same as the force law for SHM andtherefore the system executes a simple harmonicmotion. Therefore,
- **ω = √k/m**
- T = 2√m/k where T is the period.

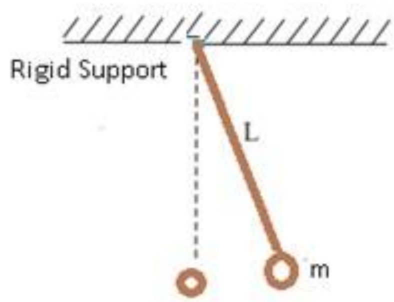
Problem: - A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?
Answer:
Maximum mass that the scale can read, M = 50 kg
Maximum displacement of the spring = Length of the scale, l = 20 cm = 0.2 m
Time period, T = 0.6 s
Maximum force exerted on the spring, F = Mg
where,g = acceleration due to gravity = 9.8 m/s^2
F = 50 × 9.8 = 490 N
∴spring constant, k = F/l = 490/0.2 = 2450 N m^{-1}.
Mass m, is suspended from the balance
∴Weight of the body = mg = 22.36 × 9.8 = 219.167 N
Hence, the weight of the body is about 219 N.

Problem: - A 5 kg collar is attached to a spring of spring constant 500 N m^{-1}. It slides without friction over a horizontal rod. The collar is displaced from its equilibrium position by 10.0 cm and released. Calculate
(a) the period of oscillation,
(b) the maximum speed and
(c) maximum acceleration of the collar.
Answer:-
The period of oscillation as given by
T =2√m/k = 2 π√ (5.0 kg/500 Nm^{-1})
= (2π/10)s

= 0.63 s
(b) The velocity of the collar executing SHM is given by,
$v(t) = -A\omega \sin(\omega t + \varphi)$
The maximum speed is given by,
$v_m = A\omega$
$= 0.1\sqrt{(k/m)}$
$= 0.1\sqrt{(500 Nm^{-1}/5kg)}$
$= 1\ ms^{-1}$
and it occurs at x = 0
(c) The acceleration of the collar at thedisplacement x (t) from the equilibrium isgiven by,
$a(t) = -\omega^2 x(t)$
$= -k/m\ x(t)$
Therefore the maximum acceleration is,
$a_{max} = \omega^2 A$
$= (500\ N\ m^{-1}/5kg)\ x0.1m$
$= 10\ m\ s^{-2}$and it occurs at the extreme positions.

Simple Pendulum
A simple pendulum is defined as an object that has a small mass (pendulum bob), which is suspended from a wire or string having negligible mass.

- Whenthe pendulum bob is displaced it oscillates on a plane about the vertical line through the support.
- Simple pendulum can be set into oscillatory motion by pulling it to one side of equilibrium position and then releasing it.



In the above image one end of a bob of mass m is attached to a string of length L and another to a rigid support executing simple harmonic motion.

Problem:-What is the length of a simple pendulum, which ticks seconds?
Answer:-
$T = \sqrt{L/g}$
By using above formula
$L = gT^2/4^2$
The time period of a simple pendulum, whichticks seconds, is 2 s. Therefore, for $g=9.8\ m\ s^{-2}$ and T = 2 s,
$L = (9.8(m\ s^{-2})\ 4(s^2))/4^2 = 1\ m$

Problem:-A cylindrical piece of cork of density of base area A and height h floats in a liquid ofdensity ρ_l. The cork is depressed slightly and then released. Show that the corkoscillates up and down simple harmonically with a period
$T = 2\pi\sqrt{h\rho/\rho_l g}$

where ρ is the density of cork. (Ignore damping due to viscosity of the liquid)

Answer:-

Base area of the cork = A
Height of the cork = h
Density of the liquid = ρ_1
Density of the cork = ρ
In equilibrium:
Weight of the cork = Weight of the liquid displaced by the floating cork
Let the cork be depressed slightly by x. As a result, some extra water of a certain volume is displaced. Hence, an extra up-thrust acts upward and provides the restoring force to the cork.
Up-thrust = Restoring force, F = Weight of the extra water displaced
F = –(Volume × Density × g)
Volume = Area × Distance through which the cork is depressed
Volume = Ax
$\therefore F = -A x \rho_1 g$ .....(i)
According to the force law:
F = kx
k = F/x
where, k is constant
$k = F/x = -A\rho_1 g$.... (ii)
The time period of the oscillations of the cork:
$T = 2\pi\sqrt{m/k}$ .... (iii)
where,
m = Mass of the cork
= Volume of the cork × Density
= Base area of the cork × Height of the cork × Density of the cork
= Ahρ
Hence, the expression for the time period becomes:

$$T=2\pi\sqrt{A h\rho/A h\rho_1 g}$$

$$T = 2\pi\sqrt{h\rho/\rho_1 g}$$

DAMPED SIMPLE HARMONIC MOTION

Damped SHM can be stated as:-

1. Motion in which amplitude of the oscillating body reduces and eventually comes to its mean position.
2. Dissipating forces cause damping.
3. Consider a pendulum which is oscillating
4. After some time we can observe that its displacement starts decreasing and finally it comes to rest.
5. This implies that there is some resistive force which opposes the motion of the pendulum. This type of SHM is known as **Damped SHM**.

Damping Force:-

- It opposes the motion of thebody.
- Magnitude of damping force is proportional to the velocity of the body.
- It actsin the opposite direction of the velocity.
- Denoted by $F_d$where d is the damping force.
  - F_d= -b v where b is a damping constant and it depends on characteristics of the medium (viscosity, for example) and the size and shape of the block.
- (-ive) directed opposite to velocity

Equation for Damped oscillations: Consider a pendulum which is oscillating.
It will experience two forces

1. Restoring force F_s = -k x
2. Damping Force F_d = -b v

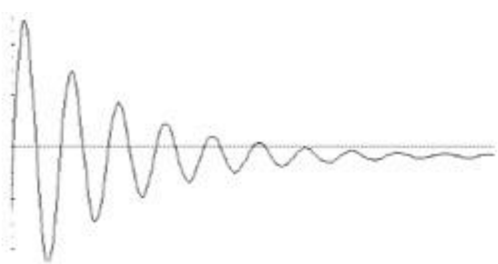
The total force F_{total} = $F_s + F_d$ = -k x – b v
Let a (t) = acceleration of the block

F_{total}= m a (t)
$-k x – b v = md^2x/dt^2$
$md^2x/dt^2 + kx + bv =0$
or $md^2x/dt^2 + b\, dx/dt + kx=0$ (v=dx/dt) (differential equation)
$d^2x/dt^2 + (b/m)\, dx/dt + (k/m)\, x=0$
After solving this equation

$x(t) = A e^{-b t/2m} \cos(\omega' t + \varphi)$ (Equation of damped oscillations)
Damping is caused by the term **$e^{-b t/2m}$**
ω' =angular frequency
Mathematically can be given as:-

$\omega' = -\sqrt{(k/m - b^2/4m^2)}$
Consider if b=0 (where b= damping force) then

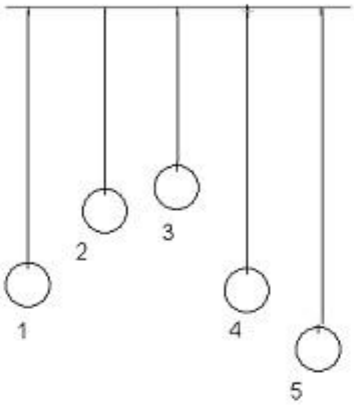
$x(t) = \cos(\omega' t + \varphi)$(Equation of Simple Harmonic motion)
Graphically if we plot Damped Oscillations



There is exponentially decrease in amplitude with time.

<u>**Free Oscillations**</u>: - In these types of oscillations the amplitude and time period remain constant it does not change. This means there is no damping. But in real scenario there is no system which has constant amplitude and time period.
<u>**Forced Oscillations**</u>: - If weapply some external force to keep oscillations continue such oscillations are known as forced oscillations.In forced oscillations the system oscillates not with natural frequency but with the external frequency.
Example of Forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations.

<u>**Resonance**</u>: -The phenomenonof increase in amplitude when the driving force is close to the natural frequency of the oscillatoris called resonance. If an external force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω, the system oscillates with angular frequency ω_d. The amplitude of oscillations is the greatest when
$\omega_d = \omega$
this expression is called resonance. Swings are very good example of resonance.



Pic: Child swinging on the swing

In the above figure there are set of 5 pendulums of different lengths suspended from a common rope.

- The figure has 4 pendulums and the strings to which pendulum bobs 1 and 4 are attached are of the same length and the others are of different lengths.
- Once displaced, the energy from this pendulum gets transferred to other pendulums through the connecting rope and they start oscillating. The driving force is provided through the connecting rope and the frequency of this force is the same as that of pendulum 1.
- Once pendulum 1 is displaced, pendulums 2, 3 and 5 initially start oscillating with their natural frequenciesand different amplitudes, but this motion is gradually damped and not sustained.
- Their oscillation frequencies slowly change and later start oscillating with thefrequency of pendulum 1, i.e. the frequency ofdriving force but with different amplitudes.
- They oscillate with small amplitudes. The oscillation frequency of pendulum 4 is different than pendulums 2, 3 and 5.
- Pendulum 4 oscillates with the same frequency as that of pendulum 1 and its amplitude gradually picks up and becomes very large.
- This happens due to the condition for resonance getting satisfied, i.e. the natural frequency of the system coincides with that of the driving force

Thank You

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