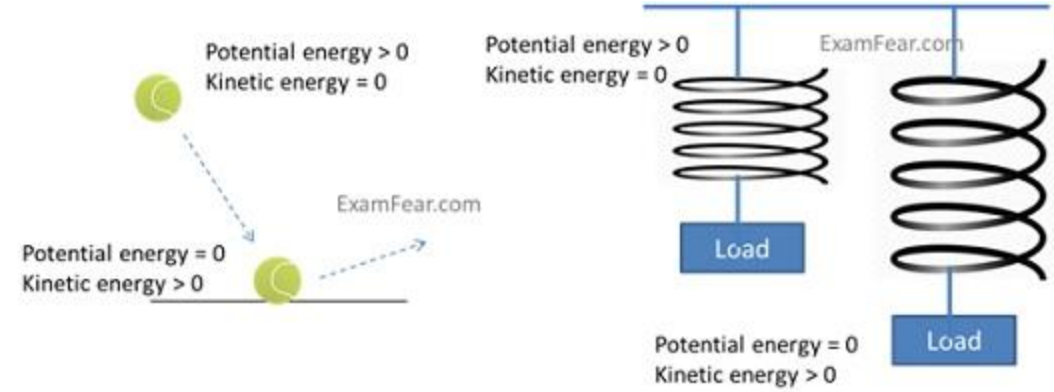
Class 12 Physics Electrostatic Potential

Introduction

The **potential energy** of a body is the energy possessed by a body by virtue of its position relative to others, stresses within it, electric charge, and other factors. Examples of mechanical potential energy are throwing a ball in air, a tightened spring etc. Work done in throwing ball and keeping spring tightened gets stored in the form of potential energy which gets converted into kinetic energy once the ball lands on the ground or the spring is released.

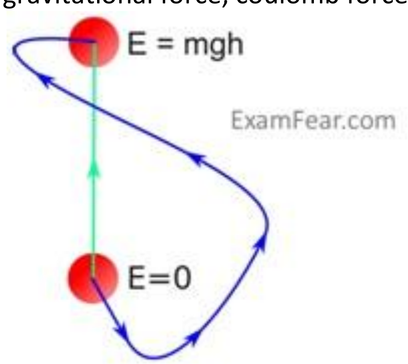


Potential energy gets converted into kinetic. The total energy always remains conserved.

Similar to the above concept of mechanical potential energy, there is a concept of electrostatic potential energy which involves charges and electric field. These concepts have been explained in the following topics.

Conservative Forces

When one form of energy gets converted to another completely on application or removal of external force, the forces are said to be **conservative**. Examples of conservative forces are sum of kinetic and potential energies working on a body, spring and gravitational force, coulomb force between two stationary charges, etc.

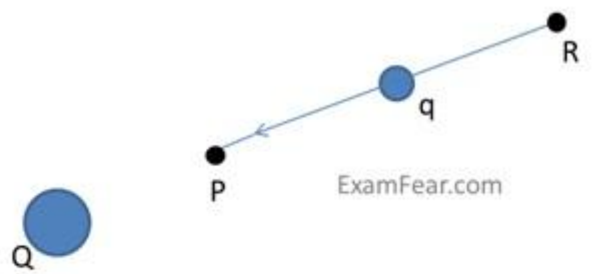


Work done by conservative gravitational force is same for different paths followed by a particle to reach from one point to another.

- Work done in moving an object from one point to another depends only on the initial and final positions and is independent of the path taken.

Electrostatic Potential

Electrostatic potential at a point is the work done by the external force in bringing a charge from infinity to that point.



A test charge q being moved from R to P against repulsive force of charge Q

To bring charge q from R to P, force required,
$F_{ext} = -F_E$ (repulsive force)

Work done,
$W_{RP} = \int_R^P F_{ext}.\,dr = -\int_R^P F_E.\,dr$

This work done will be stored in the form of potential energy. When external force is removed, the charge q moves away from Q under repulsive force. Hence the potential energy gets converted into kinetic energy. The sum therefore remains conserved.

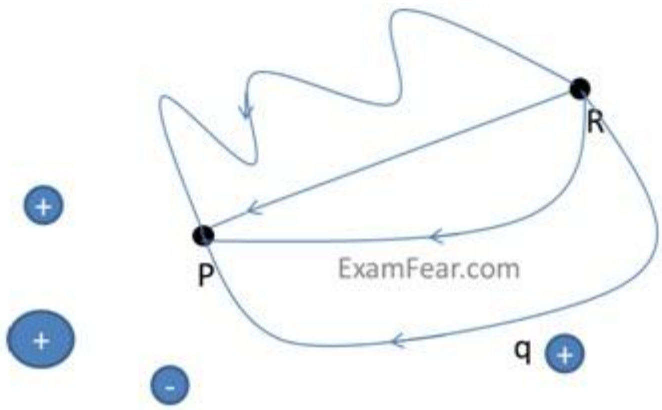
- The quantity of potential difference is more physically significant than actual value of potential energy.

$\mathbf{W_{RP} = \Delta U = U_P - U_R}$

$(U_P + \alpha) - (U_R + \alpha) = U_P - U_R$, the arbitrary α has no effect on potential energy difference

If the initial point is at infinity, $W_{\infty P} = \Delta U = U_P - U_\infty$

U_∞ is usually considered as 0, so $\mathbf{W_{\infty P} = \Delta U = U_P}$



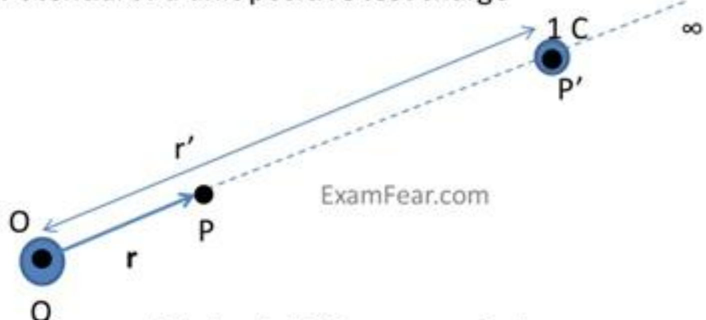
The work done to bring the charge q from R to P along any path will be same in a charge configuration.

Electrostatic Potential due to a Point Charge

The electrostatic potential in bringing a unit positive test charge from infinity to a point P against repulsive force of another positive charge is always positive.

- If the charge Q is negative, the potential is also negative as the test charge will have a force of attraction.
- Electrostatic potential is inversely proportional to r and electrostatic field is inversely proportional to square of r.

Potential of a unit positive test charge



Electrostatic force on unit charge,

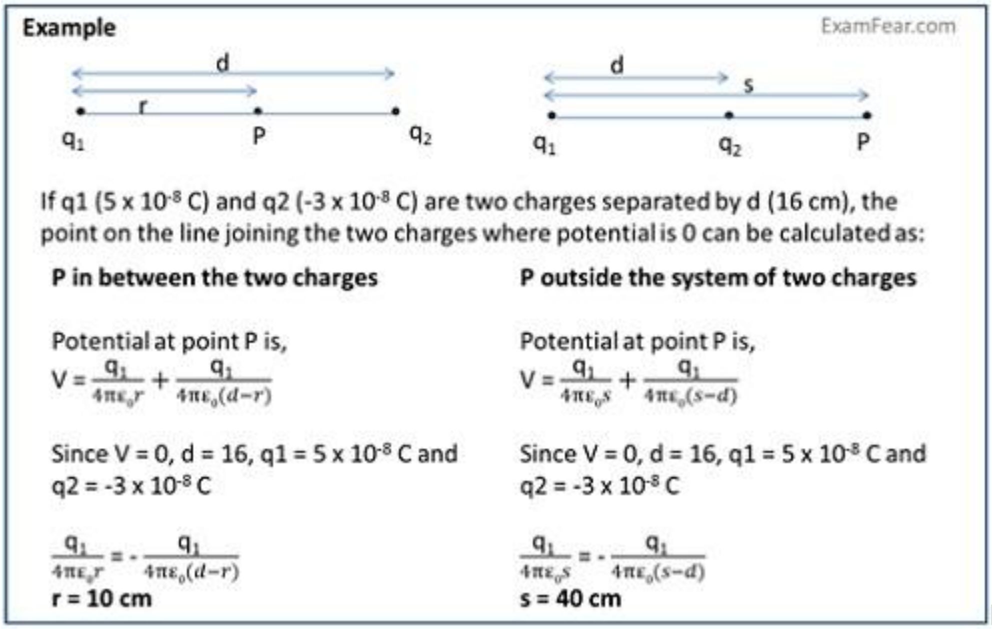
$\mathbf{F} = \frac{Q}{4\pi\varepsilon_0 r'^2}\hat{\mathbf{r}}'$

Work done against this force,

$\Delta W = -\frac{Q}{4\pi\varepsilon_0 r'^2}\Delta r'$

Total work done or potential,

$W = V(r) = \frac{Q}{4\pi\varepsilon_0 r}$



Example ExamFear.com

If q1 (5×10^{-8} C) and q2 (-3×10^{-8} C) are two charges separated by d (16 cm), the point on the line joining the two charges where potential is 0 can be calculated as:

P in between the two charges

Potential at point P is,

$V = \frac{q_1}{4\pi\varepsilon_0 r} + \frac{q_1}{4\pi\varepsilon_0 (d-r)}$

Since V = 0, d = 16, q1 = 5×10^{-8} C and q2 = -3×10^{-8} C

$\frac{q_1}{4\pi\varepsilon_0 r} = -\frac{q_1}{4\pi\varepsilon_0 (d-r)}$

r = 10 cm

P outside the system of two charges

Potential at point P is,

$V = \frac{q_1}{4\pi\varepsilon_0 s} + \frac{q_1}{4\pi\varepsilon_0 (s-d)}$

Since V = 0, d = 16, q1 = 5×10^{-8} C and q2 = -3×10^{-8} C

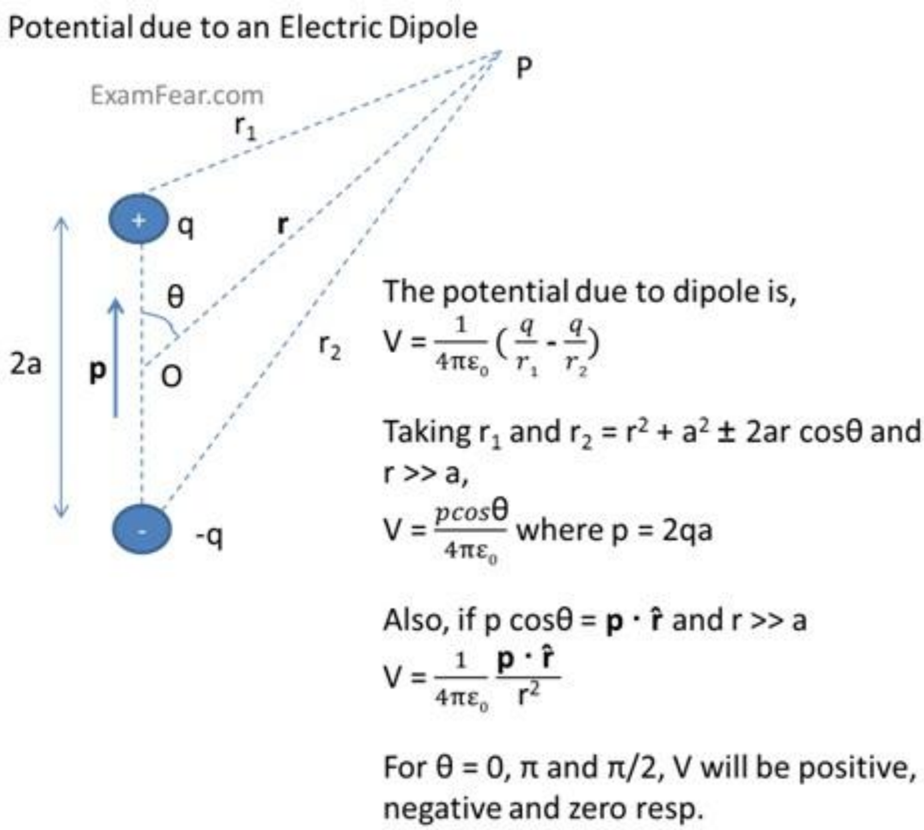
$\frac{q_1}{4\pi\varepsilon_0 s} = -\frac{q_1}{4\pi\varepsilon_0 (s-d)}$

s = 40 cm

<u>Electrostatic Potential due to an Electric Dipole</u>

An electric dipole is an arrangement of two equal and opposite charges separated by a distance 2a. The dipole moment is represented by **p** which is a vector quantity.

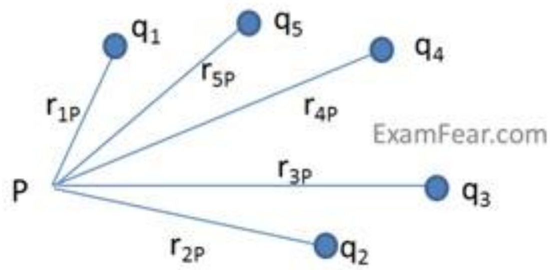
- The potential due to a dipole depends on r (distance between the point where potential is calculated and the mid-point of the dipole) and angle between position vector **r** and dipole moment **p**.
- Dipole potential is inversely proportional to square of r.



<u>Electrostatic Potential due to a system of charges</u>

For a number of charges present in space, the total potential at a point due to all those charges will be equal to the sum of individual potential of each charge at that point.

Potential due to a system of charges



Potential due to charges would be,

$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$, $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}}$, $V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{nP}}$

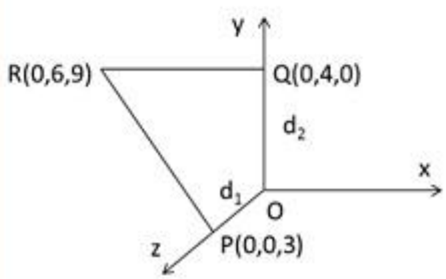
Total potential,

$V = V_1 + V_2 + ... + V_n$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + .. + \frac{q_n}{r_{nP}}\right)$$

Example ExamFear.com

If a charge q (8 mC) is located at the origin. The work done to move a charge q_1(-2×10^{-8} C) from P(0,0,3) to Q(0,4,0) via R(0,6,9) can be calculated as:



Potential at P and Q due to charge located at origin are,

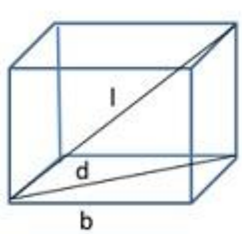
$V_1 = \frac{q}{4\pi\varepsilon_0 d_1}$ and $V_2 = \frac{q}{4\pi\varepsilon_0 d_2}$

Work done by electrostatic force to move q_1 (independent of path) will be:

$W = q_1(V_2 - V_1) = \frac{qq_1}{4\pi\varepsilon_0}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] = 1.27$ J

Example ExamFear.com

Electric potential at the center of a cube due to charges 'q' present at each vertices can be calculated as:



Diagonal of one of the six faces of the cube,

$d^2 = \sqrt{b^2 + b^2}$

$d = b\sqrt{2}$

Diagonal of the cube,

$l^2 = \sqrt{d^2 + b^2} = \sqrt{(b\sqrt{2})^2 + b^2} = \sqrt{3b^2}$

$l = b\sqrt{3}$

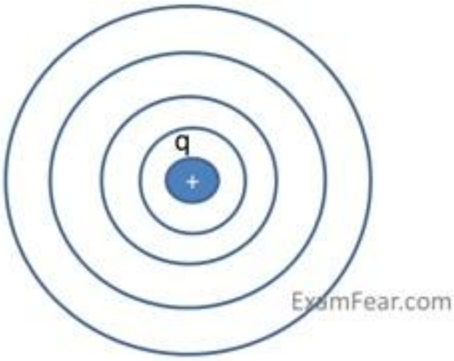
Center of the cube = r = l/2

Potential at center = $V = \frac{8q}{4\pi\varepsilon_0 r} = \frac{8q}{4\pi\varepsilon_0(b\sqrt{3}/2)} = \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$

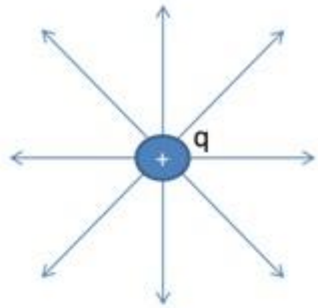
Equipotential Surfaces

An equipotential surface is one where the potential is same at every point on the surface. For a single charge,

- Since r is constant, the equipotential surfaces are concentric spherical surfaces centered at the charge.
- Electric field lines are radial starting from or ending at the charge for positive and negative charges respectively.

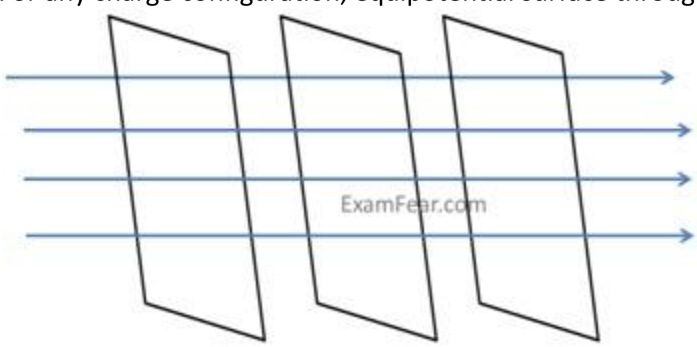


Equipotential surface for a single charge



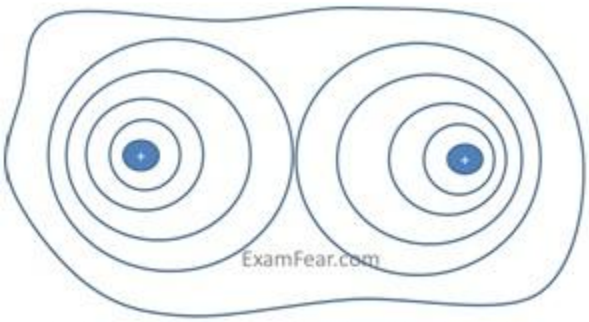
Electric field lines for a single positive charge

For any charge configuration, equipotential surface through a point is normal to the electric field at that point.



Equipotential surfaces for a uniform electric field

Equipotential surface for a dipole



Equipotential surface for two identical charges

Example

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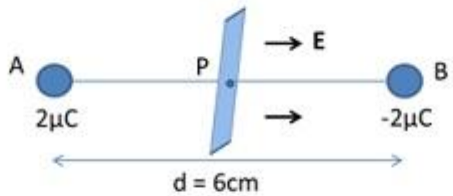


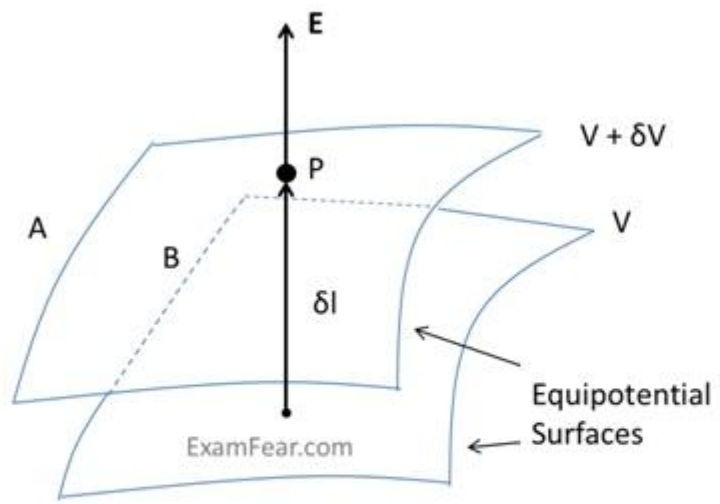
Equipotential surface for two charges 2μC and -2μC separated by 6cm will be the plane (P), located at the midpoint of line AB, normal to AB since the equipotential surface is a plane on which total potential is zero everywhere.

Also the direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

Relation between electric field and potential

According to the relation between electric field and potential,

- Electric field is in the direction in which the potential decreases steepest.
- Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.



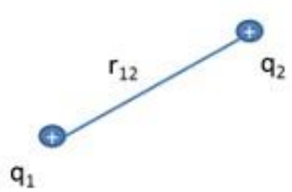
For moving unit charge along perpendicular from surface B to A,
Work done = $|\mathbf{E}|\delta l$

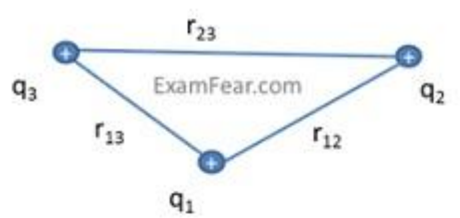
Since, Potential difference = Work done
$|\mathbf{E}|\delta l = -\delta V$
$|\mathbf{E}| = -\delta V/\delta l = +|\delta V|/\delta l$

<u>Potential Energy of a system of charges</u>

The potential energy is characteristic of the current state of configuration and not the way how this configuration was achieved. The potential for a system of charges is calculated as below:

- Potential energy for the first charge is considered as zero which is brought from infinity to a point having no external repulsive forces (no other charges present in the space).
- Bringing another charge will face attractive or repulsive force from the first charge and the work done to bring it from infinity to the point is stored in the form of potential energy.
- Similarly, all further charges being brought from infinity will face attractive or repulsive force from all other charges present in the space.





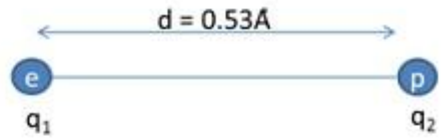
Potential energy of a system of 2 charges,

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Potential energy of a system of 3 charges,

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Example ExamFear.com



Potential due to a hydrogen atom whose proton and electron are separated by 0.53Å having charge 1.6×10^{-19} C:

$$\text{Since, } U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} \text{ and } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$

$U = 9 \times 10^9 \times (1.6 \times 10^{-19})^2 / 0.53 \times 10^{-10}$
$U = -43.7 \times 10^{-19}$ J or Joules
Converting the above result in electron-Volt),

$U = -43.7 \times 10^{-19} / 1.6 \times 10^{-19}$ eV (electron-Volt)
U = -27.2 eV

Potential Energy of point charge in an external field

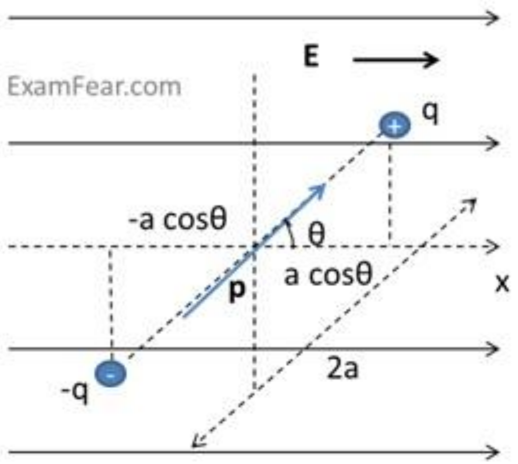
When the potential energy of charge(s) is calculated in an external field, following things are understood:

- The electric field, E, is produced by external sources which are fixed and may be unknown.
- The charge q does not affect these external sources.
- Potential energy of the external sources is ignored.

Potential Energy of a single charge	**Potential energy of a system of two charges**
$qV(\mathbf{r})$ $\mathbf{r}$– position vector $V(\mathbf{r})$ – external potential at point $\mathbf{r}$	$q_1V(\mathbf{r}_1) + q_2V(\mathbf{r}_2) + q_1q_2/(4\pi\varepsilon_0 r_{12})$ $\mathbf{r}_1$&$\mathbf{r}_2$– position vectors $V(\mathbf{r}_1)$ & $V(\mathbf{r}_2)$ – external potential at point $\mathbf{r}_1$&$\mathbf{r}_2$

Potential Energy of a dipole in an external field

Dipole in an external field experiences a torque which tries to align it in the direction of the electric field. The work done to oppose this force gets stored in the form of potential energy.



In a uniform electric field, dipole experiences a torque,
$\tau = \mathbf{p} \times \mathbf{E}$

Work done by external torque against the existing torque,
$W = pE(\cos\theta_0 - \cos\theta_1)$

Taking θ_0 as 90° and θ_1 as θ, Potential energy,
$U(\theta) = -\mathbf{p}\cdot\mathbf{E} = -pE\cos\theta$

Example ExamFear.com

The molecules of a substance have a permanent electric dipole moment of 10^{-29} C m. A mole of this substance is polarized by an electrostatic field. If the electrostatic field tilts at an angle of 60°, the change in potential energy to align the dipole in the direction of electric field can be calculated as:

Dipole moment of each molecule = 10^{-29} C m
1 mole = 6×10^{23}
Total dipole moment = $6 \times 10^{23} \times 10^{-29} = 6 \times 10^{-6}$ C m
Initial potential energy = $U_i = -pE\cos\theta = -6 \times 10^{-6} \times 10^{-6}\cos 0 = -6$ J
Final potential energy = $U_f = -pE\cos\theta = -6 \times 10^{-6} \times 10^{-6}\cos 60 = -3$ J
Change in potential energy = $-6 - (-3) = 3$ J

Thank You