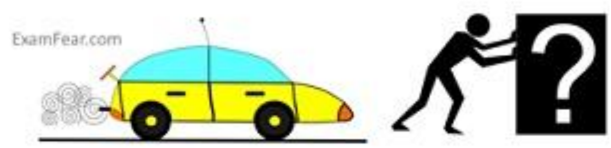
Scalars Vs. Vectors

Criteria	**Scalar**	**Vector**
Definition	A scalar is a quantity with magnitude only.	A vector is a quantity with magnitude and direction.
Direction	No	Yes
Specified by	A number (magnitude) and a unit	A number (magnitude), direction and a unit
Represented by	quantity's symbol	quantity's symbol in bold or an arrow sign above
Example	mass, temperature	velocity, acceleration

Scalar Quantities – Mass and Temperature

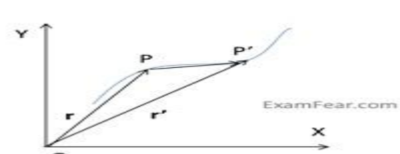


Vector Quantities – Velocity and Force

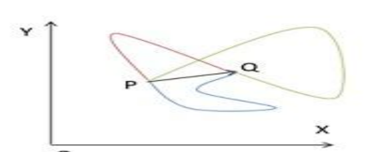
Position and Displacement Vectors

Position Vector: Position vector of an object at time t is the position of the object relative to the origin. It is represented by a straight line between the origin and the position at time t.

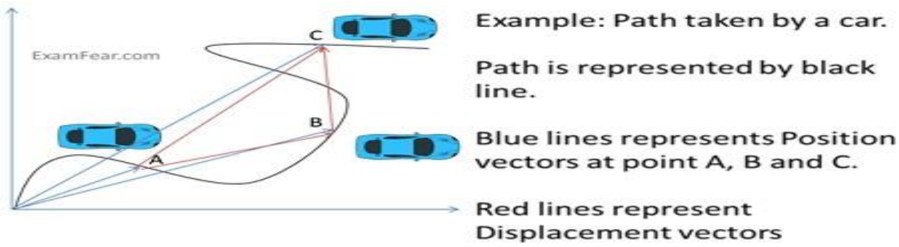
Displacement Vector: Displacement vector of an object between two points is the straight line between the two points irrespective of the path followed. The path length is always equal or greater than the displacement.

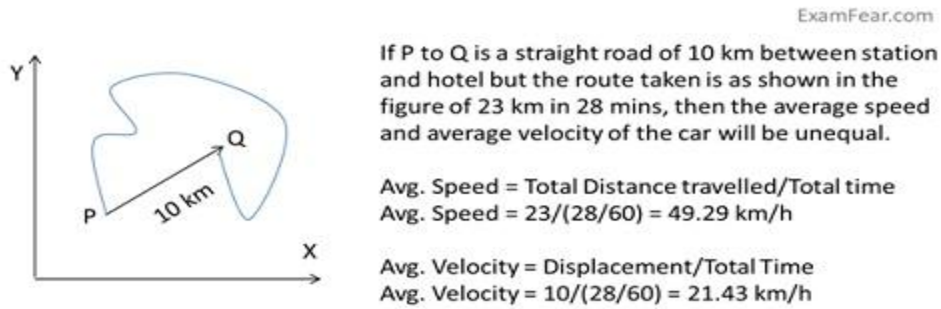


OP and OP' are position vectors represented by r and r'.
PP' is the displacement vector.



PQ is the displacement vector for any path followed (represented by green, blue and red paths).

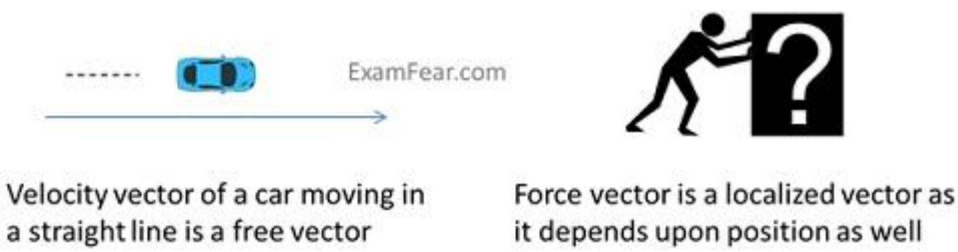




Free and Localized Vectors

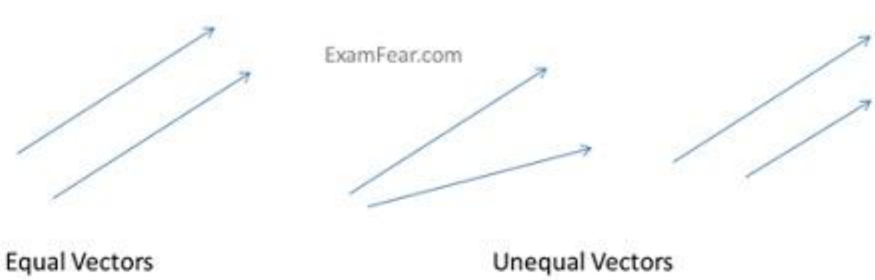
A **free vector**(or non-localized vector) is a vector of which only the magnitude and direction are specified, not the position or line of action. Displacing it parallel to itself leaves it unchanged.

A **localized vector** is a vector where line of action and position are as important as magnitude and direction. These vectors change with change in position and direction.



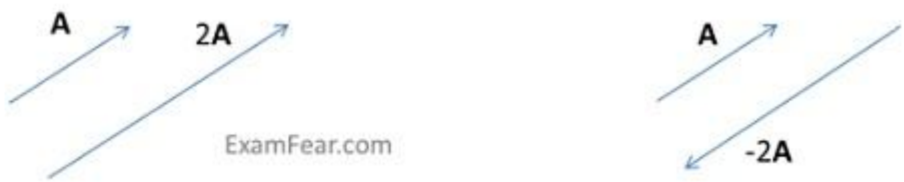
Equality of Vectors

Two vectors are said to be equal only when they have same direction and magnitude.For example, two cars travelling with same speed in same direction. If they are travelling in opposite directions with same speed, then the vectors are unequal.



Multiplication of Vectors with real numbers

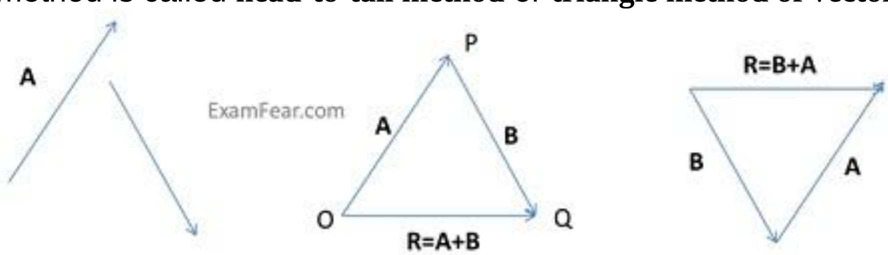
Multiplication Factor	Original vector	Magnitude of vector after multiplication	Direction of vector after multiplication
λ (>0)	**A**	**λA**	Same as that of A
-λ (<0)	**A**	**λA**	Opposite to that of A
λ (=0)	**A**	**0 (null vector)**	None. The initial and final positions coincide.



Multiplication of vector with +2 and -2

Addition and Subtraction of Vectors – Triangle Method

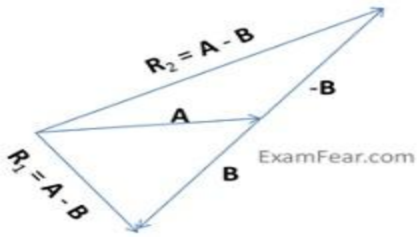
The method of adding vectors graphically is by arranging them so that head of first is touching the tail of second vector and making a triangle by joining the open sides. This method is called **head-to-tail method** or **triangle method of vector addition**



Head-to-Tail or Triangle Method of vector addition

- Vector addition is:
  - Commutative: **A** + **B** = **B** + **A**
  - Associative: (**A** + **B**) + **C** = **A** + (**B**+ **C**)
- Adding two vectors with equal magnitudes and opposite directions results in null vector.
  - Null Vector: **A** + (-**A**) = **0**
- Subtraction is adding a negative vector(opposite direction) to a positive vector.

- **A – B = A + (-B)**

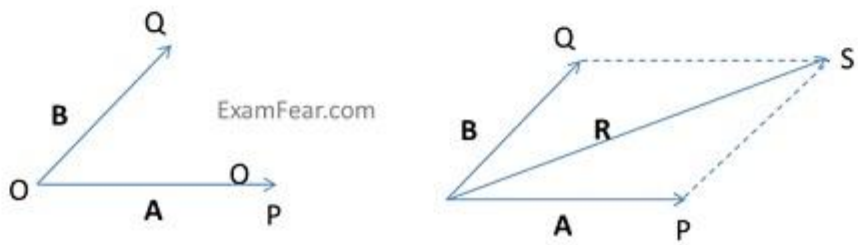


Addition and Subtraction of vectors using triangle method

Addition of Vectors – Parallelogram Method

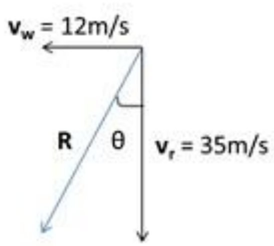
The method of adding vectors by parallelogram method is by:

- Touching the tail of the two vectors
- Complete a parallelogram by drawing lines from the heads of the two vectors.
- Vector resulting from the origin to the point of intersection of above lines gives the addition.



Parallelogram Method of vector addition

Example ExamFear.com



If rain is falling vertically at a speed of 35 m/s and wind is blowing at 12 m/s (east to west), then the resultant vector **R** would be the actual path of rain.

$R = \sqrt{v_r^2 + v_w^2} = \sqrt{35^2 + 12^2} = 37 m/s$

$\tan\theta = v_w/v_r = 12/35 = 0.343$

$\theta = 19°$

A person standing on ground must hold umbrella at an angle of 19° with vertical towards east to avoid rain.

Resolution of Vectors

A vector can be expressed in terms of other vectors in the same plane. If there are 3 vectors **A**, **a** and**b**, then **A** can be expressed as sum of **a** and **b** after multiplying them with some real numbers.

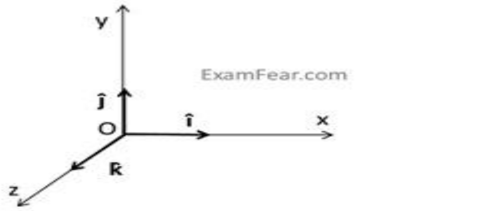
3 vectors in a plane Joining the three to make a triangle

A can be resolved into two component vectors λ**a** and μ**b**. Hence, **A** = λ**a** + **μ**b. Here λ and μ are real numbers.

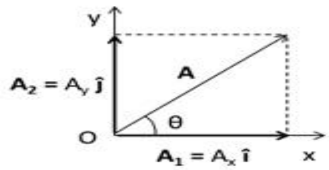
Unit Vectors

A **unit vector** is a vector of unit magnitude and a particular direction.

- They specify only direction. They do not have any dimension and unit.
- In a rectangular coordinate system, the x, y and z axes are represented by unit vectors, $\hat{i}$,$\hat{j}$ and$\hat{k}$
- These unit vectors are perpendicular to each other.
- $|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$



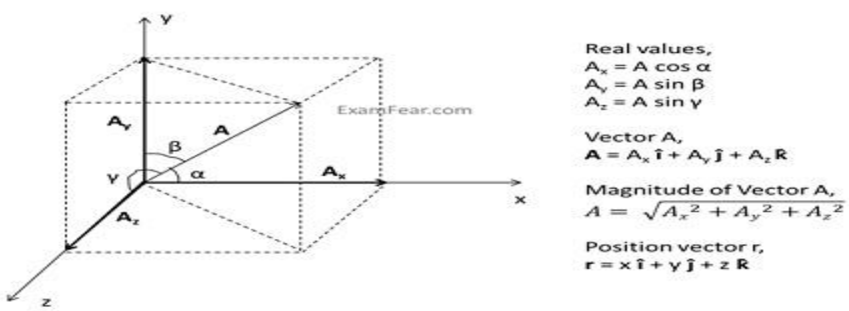
Unit vectors in the coordinate system along the three axes.



Vector **A** as combination of **A₁** and **A₂** which are expressed in terms of unit vectors.

In a 2-dimensional plane, a vector thus can be expressed as:

1. $\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$ where, $A_x = A \cos\theta$ and $A_y = A \sin\theta$
2. $A = \sqrt{A_x^2 + A_y^2}$

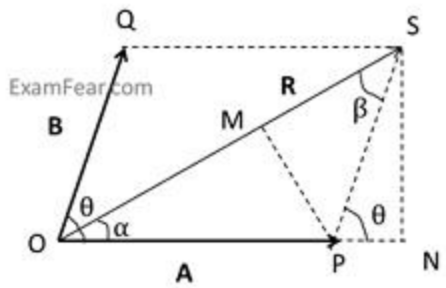


Vector resolved along the three axes

Analytical Method of Vector Addition

Vectors	**Sum of the vectors**	**Subtraction of the vectors**
$\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}}$ and $\mathbf{B} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}}$	$\mathbf{R} = \mathbf{A} + \mathbf{B}$ $\mathbf{R} = R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}}$where $R_x = A_x + B_x$and$R_y = A_y + B_y$	$\mathbf{R} = \mathbf{A} - \mathbf{B}$ $\mathbf{R} = R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}}$where $R_x = A_x -$ $B_x$and$R_y = A_y - B_y$
$\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{k}}$ $\mathbf{B} = B_x \hat{\mathbf{i}} + B_y \hat{\mathbf{j}} + B_z \hat{\mathbf{k}}$	$\mathbf{R} = \mathbf{A} + \mathbf{B}$ $\mathbf{R} = R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}} + R_z \hat{\mathbf{k}}$where $R_x = A_x + B_x$and$R_y = A_y + B_y$ and$R_z = A_z + B_z$	$\mathbf{R} = \mathbf{A} - \mathbf{B}$ $\mathbf{R} =$ $R_x \hat{\mathbf{i}} + R_y \hat{\mathbf{j}} + R_z \hat{\mathbf{k}}$where $R_x = A_x -$

		B_x and $R_y = A_y$ - B_y and $R_z = A_z - B_z$



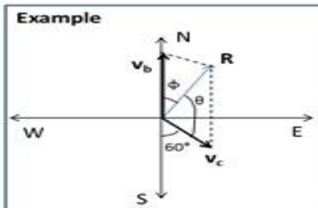
Law of Cosines

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

Law of Sines

$$\frac{R}{\sin\theta} = \frac{A}{\sin\beta} = \frac{B}{\sin\alpha}$$

Example

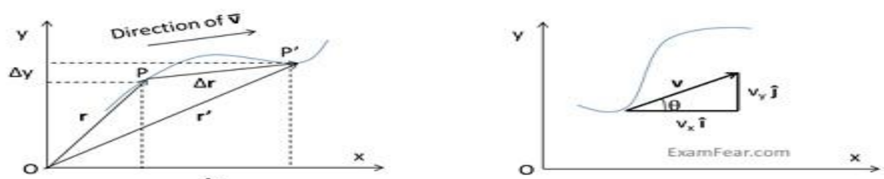


If a motorboat is travelling at a speed of 25 km/h north and water current is 10 km/h at an angle of 60° (east to south), then the resultant velocity and direction of boat can be obtained using law of sines and cosines.

Magnitude of R using law of cosines,
$R = \sqrt{v_b^2 + v_c^2 + 2v_bv_c\cos 120°} = 22$ km/h

Direction of R using law of sines,
$R/\sin\theta = v_c/\sin\phi$, $\sin\phi = v_c\sin\theta/R$
$\phi = 23.4°$

<u>Quantities related to motion of an object in a plane</u>



Particle moving in a plane from P to P' in time t to t' and Velocity calculation of the particle in terms of unit vectors

Quantity	Value	Value in component form
Displacement Δr (Change in position)	**r' - r**	**î**Δx + **ĵ**Δy
Average Velocity v̄ (ratio of displacement and corresponding time interval)	Δ**r**/Δ**t**	v_x**î** + v_y **ĵ** $v_x = \Delta x/\Delta t$, $v_y = \Delta y/\Delta t$
Instantaneous velocity v (limiting value of average velocity as the time interval approached zero)	d**r**/dt	v_x**î** + v_y **ĵ** $v_x = dx/dt$, $v_y = dy/dt$

Magnitude of **v**		
Direction of **v,** θ (direction of velocity at any point on the path is tangential to the path at that point and is in the direction of motion)	$\tan^{-1}(v_y/v_x)$	
Average Acceleration $\bar{\mathbf{a}}$ (change in velocity divided by the time interval)	$\Delta\mathbf{v}/\Delta t$	$a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$ $a_x = \Delta v_x/\Delta t$, $a_y = \Delta v_y/\Delta t$
Instantaneous acceleration $\mathbf{a}$ (limiting value of the average acceleration as the time interval approaches zero)	$d\mathbf{v}/dt$	$a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}}$ $a_x = dv_x/dt$, $a_y = dv_y/dt$ $a_x = d^2x/dt^2$, $a_y = d^2y/dt^2$

Motion in a plane with constant acceleration

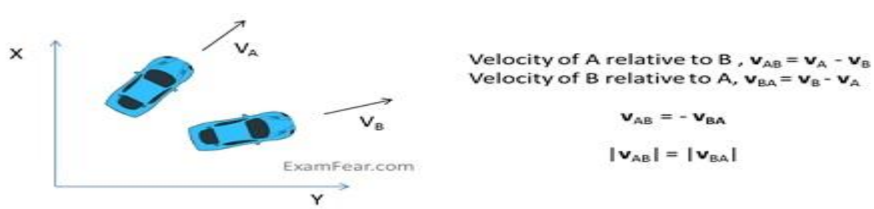
Motion in a plane (two dimensions) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions. X and Y directions are hence independent of each other.

If v_0 being the velocity at time 0, the displacement can be written as:
$x = x_0 + v_{0x}t + ½\, a_x t^2$ and $y = y_0 + v_{0y}t + ½\, a_y t^2$

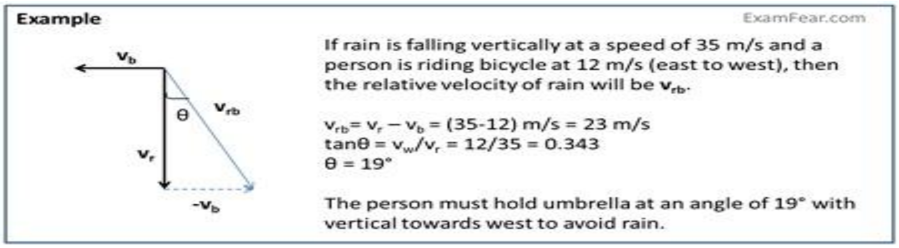
Motion of an object in a plane with constant acceleration		
Velocity	**Velocity in terms of components**	**Displacement**
$\mathbf{v} = \mathbf{v_0} + \mathbf{a}t$	$v_x = v_{0x} + a_x t$ $v_y = v_{0y} + a_y t$	$\mathbf{r} = \mathbf{r_0} + \mathbf{v_0}t + ½\, \mathbf{a}t^2$

Relative velocity in two dimensions

The concept of relative velocity in a plane is similar to the concept of relative velocity in a straight line.



Relative Velocity in a plane

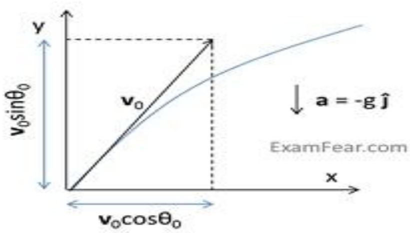


Projectile Motion

An object that becomes airborne after it is thrown or projected is called **projectile**. Example, football, javelin throw, etc.

Projectile Motion

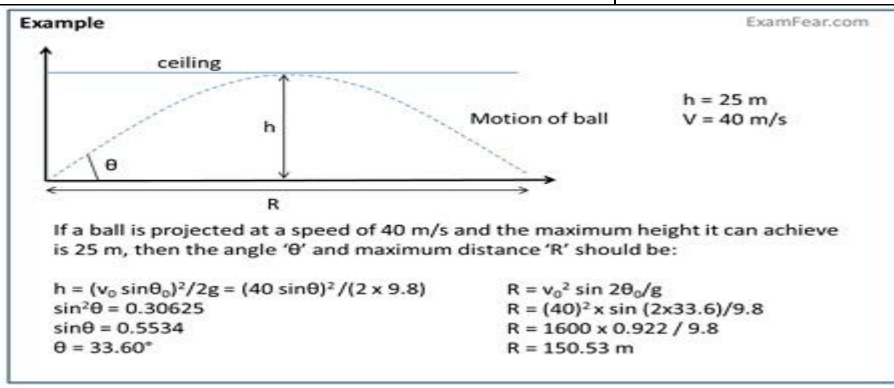
- Projectile motion comprises of two parts – horizontal motion of no acceleration and vertical motion of constant acceleration due to gravity.
- Projectile motion is in the form of a parabola, $y = ax + bx^2$.
- Projectile motion is usually calculated by neglecting air resistance to simplify calculations.



Motion of an object projected with velocity $\mathbf{v_0}$ at an angle θ_0

Quantity	**Value**
Components of velocity at time t	$v_x = v_0 \cos\theta_0$ $v_y = v_0 \sin\theta_0 - gt$
Position at time t	$x = (v_0 \cos\theta_0)t$ $y = (v_0 \sin\theta_0)t - \frac{1}{2} gt^2$
Equation of path of projectile motion	$y = (\tan \theta_0)x - gx^2/2(v_0 \cos\theta_0)^2$
Time of maximum height	$t_m = v_0 \sin\theta_0 /g$

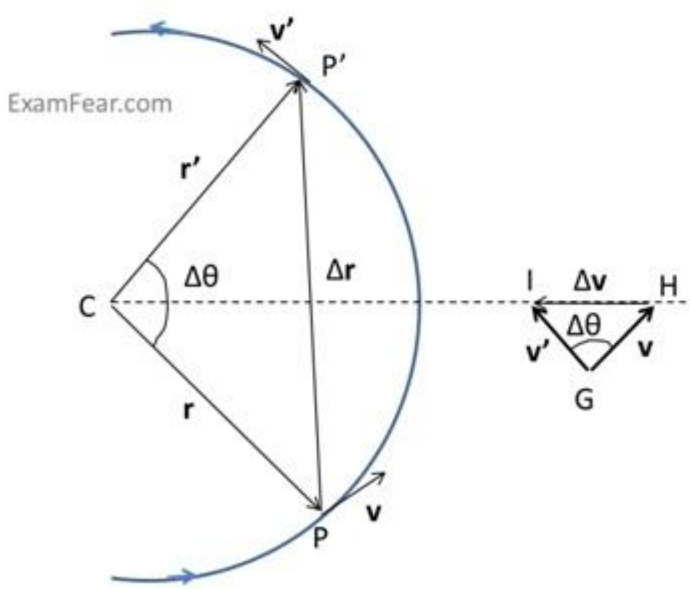
Time of flight	$2\ t_m = 2\ (v_0 \sin\theta_0 /g)$
Maximum height of projectile	$h_m = (v_0 \sin\theta_0)^2/2g$
Horizontal range of projectile	$R = v_0^2 \sin 2\theta_0/g$
Maximum horizontal range (θ_0=45°)	$R_m = v_0^2/g$



Uniform circular motion

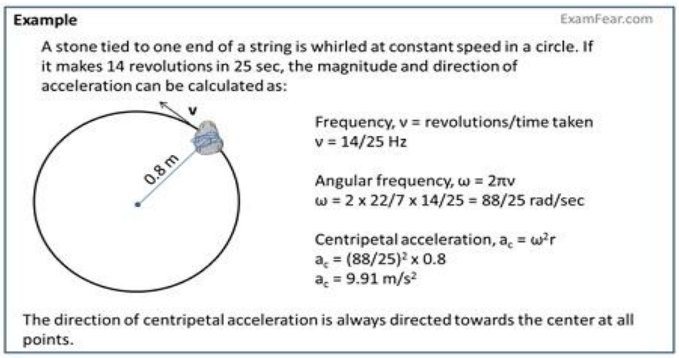
When an object follows a circular path at a constant speed, the motion is called **uniform circular motion**.

- Velocity at any point is along the tangent at that point in the direction of motion.
- Average velocity between two points is always perpendicular to Average displacement. Also, average acceleration is perpendicular to average displacement.
- For an infinitely small time interval, Δtà 0, the average acceleration becomes instantaneous acceleration which means that in uniform circular motion the acceleration of an object is always directed towards the center. This is called **centripetal acceleration**.



Velocity and Acceleration of an object in uniform circular motion

Quantity	Values
Centripetal Acceleration	$a_c = v^2/R$, R – radius of the circle $a_c = \omega^2R$, ω – angular speed $a_c = 4\pi^2v^2R$, v – frequency
Angular Distance	$\Delta\theta = \omega\ \Delta t$
Speed	$v = R\omega$



Thank You

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