Wisdom Education Academy

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INTRODUCTION

External force is needed to make a stationary body move as well as to stop a moving body.

To understand this chapter i.e. Laws of Motion, lets have a quick recap on what we already know about Motion.

- Motion in simple is tendency to move. An object is said to be in motion when it changes its position with time
- A body is said to be at Rest if it is not moving
- An external force is always involved in motion. This force helps body to perform required actions and these actions are according to rules or laws i.e. Laws of Motion.

Contact & Non-contact forces

- External force comes into picture when the body starts moving or comes to rest either by coming in contact with the body or without being in contact.

- **Contact force**- force applied by coming in contact with the body

Example: hitting of cricket ball with bat in game

- **Non-contact force**- force applied without coming in contact with body
  Example: coin attracted towards magnet(magnetic force)

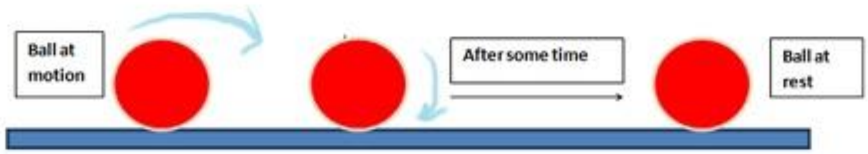
  ball dropped at height attracted towards the earth (gravitational force)

Aristotle's fallacy:

Aristotle, the Greek Scientist held the view that an external force is required to keep a body in uniform motion.

His concept is **obsolete now**, because he considered only one side of motion and fails to explain the other i.e. if body is in motion then how does it come to rest? There came the concept of the opposing external force of Friction.

For example, a ball rolled on the floor comes to rest after some time due to opposing force of friction.



Conclusion:

An external force is required to keep a body in motion, only if resistive forces (like frictional & viscous forces) are present.

Inertia

Inertia is the resistance of a body to change its state of motion.

- A body at rest wants to be at rest
- A body in motion wants to be in motion

Have you experienced jerk when brakes are suddenly applied to the vehicle?

This happens because initially the car was in motion; the body was also in motion. Later, due to the application of brakes, the car came to rest; but the body due to its inertia still wants to be in motion. Therefore, the body doesn't come to rest at once and experiences jerk.

Galileo's Law of Inertia

- Aristotle's Law was falsified by Galileo. Galileo corrected Aristotle's law saying "An external force is required to keep a body in motion, only if resistive forces are present"

- According to this law, A body moving on a frictionless surface should move with constant velocity.
- A body at rest is considered equivalent to a body in uniform motion (In both cases, acceleration = 0; net force = 0)
- Galileo's law is the basis of Newton's 1st Law of motion

Newton's First law of motion

- A body at rest tends to remain at rest and a body in uniform motion tends to remain in the state of uniform motion until & unless an external force is applied on it.

For example, a ball lying on the table at rest will remain at rest until an external force is applied on it.

Balanced & Unbalanced forces

Balanced Forces:

- Equal & Opposite forces
- Do not cause any change in motion

Unbalanced Forces:

- Unequal forces
- Can be in the same or opposite direction
- Causes a change in motion

For example, in Tug of war, If teams 1 & 2 apply equal forces in opposite directions, there would be no net force. This is Balanced force.

However, if Team 1 exerts more force than Team 2, then there would be a net movement towards Team 1 and Team 1 would win. This is unbalanced force.

Momentum

Newton's first law was for scenarios where net force = 0. The second law is for scenarios with net force not equal to 0. Momentum plays a crucial role in Second law.

- Momentum is the product of mass of a body & its velocity
- It is a Vector quantity
- It is denoted by $p = mv$

For example, A ball of 1 kg moving with 10m/sec has a momentum 10kg m/sec.
Momentum of a system remains conserved. Therefore,

- Greater force is required to set heavier bodies in motion

- Greater force is required to stop bodies moving with higher velocities

Conclusion: Greater the change in momentum in a given time, greater is the force that needs to be applied. In other words, greater the change in momentum vector, greater is the force applied.

Newton's Second Law

- The rate of change of momentum of a body is directly proportional to the applied force and takes place in the direction in which the force acts.
- Alternatively, the relationship between an object's mass m, its acceleration a, and the applied force F is $F = ma$; the direction of the force vector is the same as the direction of acceleration vector.

$F \propto dp/dt$ [Greater the change in momentum, greater is force]
$F = k\, dp/dt$
$F = dp/dt$
$F = d/dt\,(mv)$
Let, m: mass of the body be constant
$F = m\, dv/dt$
$F = ma$

- Newton's Second law is consistent with the First law

$F = ma$
If $F = 0$, then $a = 0$
According to First law, if $a = 0$, Then $F = 0$
Thus, both the laws are in sync.

- Vector form of Newton's Second law

$$\vec{F} = \vec{F_x} i + \vec{F_y} j + \vec{F_z} k$$

$F_x = dp_x/dt = ma_x$
$F_y = dp_y/dt = ma_y$
$F_z = dp_z/dt = ma_z$

- Newton's Second Law was defined for point objects. For larger bodies,
  - a: acceleration of the centre of mass of the system.
  - F: total external force on the system.

Problem 1: A car of mass $2 * 10^3$ kg travelling at 36 km/hr on a horizontal road is brought to rest in a distance of 50m by the action of brakes and frictional forces. Calculate: (a) average stopping force , (b) Time taken to stop the car

Solution.
$m = 2 * 10^3$
$u = 36$ km/hr $= 10$m/s
$s = 50$m
$v = 0$
To find: a, F, t

Third equation of motion: $v^2 = u^2 + 2as$
0 = 100 + 2a * 50
Or, **a = -1 m/s²**
Therefore, F = ma = (2 * 10^3) * 1 = **2 * 10^3 N**
V = u + at
0 = 10 – t
t = 10 sec

Problem 2: The only force acting on a 5kg object has components F_x = 15N and F_y = 25N. Find the acceleration of the object.
Solution.
m= 5kg
Fx = 15N, Fy = 25N
F = Fxi + Fyj

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2}$$

= √ 225 + 625 = √850
F = ma
Or, a = F/ m = √850/5 = 5 √34/5 = √34 = **5.83m/s²**

<u>Impulse</u>

- Impulse is defined as a force multiplied by time it acts over.
- For example: Tennis racket strikes a ball, an impulse is applied to the ball. The racket puts a force on the ball for a short time period.

F ∆t = ∆p
F = ∆p/ ∆t = Rate of Change of momentum

<u>Newton's Third Law</u>
To every action, there is always an equal and opposite reaction.
For example, when you hold the ball, a force acts on the ball (Action), and an equal and opposite force acts on your hand (Reaction).

- Action & Reaction forces always act on different bodies
  - $F_{AB} = F_{BA}$
  - F_{AB}: Force acting on A by B
  - F_{BA}: Force acting on B by A
- Action & Reaction forces occur at the same instant

Conservation of Momentum

- In an isolated system, the total momentum is conserved.

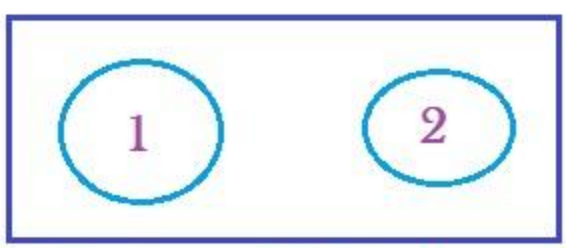
Example 1. In a Spinning top, total momentum = 0. For every point, there is another point on the opposite side that cancels its momentum.

Example 2. Bullet fired from a Rifle
Initially, momentum = 0
Later, the trigger is pulled, bullet gains momentum in àdirection, but this is cancelled by rifle's ß momentum. Therefore, total momentum = 0

During the process, the chemical energy in gunpowder gets converted into heat, sound and chemical energy.
Example3. Rocket propulsion
Initially, mass of rocket: M. It just started moving with velocity v
Initial momentum = Mv

Later, gases are ejected continuously in opposite direction with a velocity relative to rocket in downward direction giving a forward push to the rocket.
Mass of the rocket becomes (M-m)
Velocity of the rocket becomes (v + v')
Final momentum = (M –m) (v + v')
Thus, Mass * velocity = constant

Collision of Bodies



System of 2 bodies

Let the two bodies 1 & 2 have momentum p_1 & p_2 before they collided with each other. After collision their momentum are p_1' and p_2' respectively.
By Newton's Second law,

F = dp/dt

For 1: $F_{12} = (p_1' - p_1)/\Delta t$
For 2: $F_{21} = (p_2' - p_2)/\Delta t$

By Newton's Third law,
F 12 = - F21
$(p_1' - p_1)/\Delta t = (p_2' - p_2)/\Delta t$
$(p_1' - p_1) = (p_2' - p_2)$
$p_1' + p_2' = p_1 + p_2$
Conclusion: Final momentum of the system = Initial momentum of the system

Problem: A railway truck A of mass 3 * 10^4 kg travelling at 0.6 m/s collides with another truck B of half its mass moving in the opposite direction with a velocity of 0.4 m/s. If the trucks couple automatically on collision, find the common velocity with which they move.
Solution.
m_1 = 3 * 10^4 kg
m_2 = ½ of mass of A = 1.5 * 10^4 kg
u_1 = 0.6 m/s
u_2 = -0.4 m/s
Before collision: $m_1u_1 + m_2u_2$ = 3 * 10^4 * 0.6 + 1.5 * 10^{4} * (-0.4)
= 1.2 * 10^4 kg m/s
After collision: $(m_1 + m_2)$ v = 4.5 * 10^4 v kg m/s
As per conservation of momentum,
1.2 * 10^4 = 4.5 * 10^4 v
V = 1.2/ 4.5 = 0.27 m/s

Therefore, the common velocity is 0.27m/s

Equilibrium of a Particle

- A particle is said to be in equilibrium when the net external force on it is zero

- In general, $F_1 + F_2 + F_3 + \text{------} + F_n = 0$

Friction

- Friction is a contact force that opposes **relative motion**.
- No friction exists till an external force is applied.

There are 3 types of Friction:

- Static friction
  - Force that resists initiation of motion of one body over another with which it is in contact
  - Opposes Impending motion
  - Denoted by f_s

Let the ball be at rest initially

Applied force, $F_a = 0$; Static friction, $f_s = 0$

Later, applied force, $F_a = F$, then fs also increases but only up to a certain limit. As soon as F_a becomes greater than fs, the ball starts to move.

f_s acts when a body is at rest. Hence called Static friction

- Limiting value of f_s depends on Normal reaction, and is independent of the area of contact

$f_s\,max \propto N$
$f_s\,max = constant * N$
$f_s\,max = \mu_s N$
where μ_s is the coefficient of static friction
This coefficient depends on the nature of surfaces in contact.

- According to the Law of Static friction, Static friction is always less than or equal to the limiting value of f_s

$f_s =< f_s\,max$
$f_s\,max = \mu_s N$
$f_s =< \mu_s N$

Kinetic friction

- Force that resists motion of one body over another with which it is in contact.
- Denoted by f_k
- As motion starts, f_s vanishes and f_k appears

- Kinetic friction is Independent of the area of contact and velocity of the body
- It varies with Normal reaction, N

$f_k \propto N$
$f_k = constant * N$
$f_k = \mu_k N$; μ_k is the coefficient of kinetic friction
Three scenarios can arise in a body's motion

1. When applied force > f_k
   $F_a > f_k$
   $(F_a – f_k) = ma$
   $a = (F_a – f_k)/m$
2. When applied force = f_k
   $F_a = f_k$
   Therefore, a = 0 i.e. the body moves with uniform velocity

3. When applied force = 0
$F_a = 0$
$A = -f_k/m$
No motion occurs, the body stops

<u>Relation between Coefficient of Static & Kinetic friction</u>
$f_k = \mu_k N$
$f_s =< \mu_s N$
$f_s > f_k$ (to keep the body moving)
$\mu_s N > \mu_k N$
Or, $\mu_k < \mu_s$
Coefficient of kinetic friction is smaller than the coefficient of static friction.

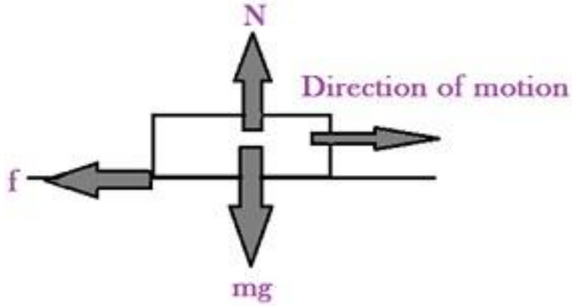
Rolling friction

- Rolling friction is applicable for bodies whose point of contact keeps changing
- It is the force that opposes the motion of a body which is rolling over the surface of another
- Bowling balls, rotating wheels are examples illustrating Rolling friction

- Coefficient of Rolling friction is lesser than that of Kinetic friction.

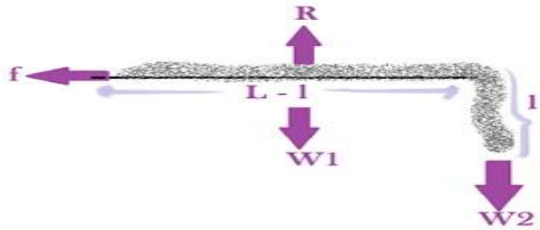
$f_r < f_k < f_s$

Problem: Calculate the force required for pushing a 30 kg wooden bar over a wooden floor at a constant speed. Coefficient of friction of wood over wood = 0.25
Solution.
M = 30 kg
$\mu = 0.25$
$(F_a - f) = ma$
For constant speed, a = 0
So, $F_a = f = \mu N$



From free body diagram, N = mg
Therefore, $F_a = f = \mu N = \mu mg = 0.25 * 30 * 9.8 = 73.5$ N

Problem 2: A homogenous chain of length L lies on a table. What is the maximum length l of the part of the chain hanging over the table if the coefficient of friction between the chain and the table is u, the chain remaining at rest with the table?
Solution.



Let ρ be the mass per unit length of the chain
P = m/l or, m = ρl
Weight of hanging part = W2 = ρlg
Weight of chain over the table = W1 = ρ(L-l)g
For equilibrium,
R = ρ(L-l)g --------------- (i)
f= W2 = ρlg
μR = ρlg -------------------(ii)
(i)Divided by (ii) gives
1/μ = (L-l)/l
l = μL – μl
l + μl = μL
Therefore, l = μL/(μ + 1)

Friction- A boon or Bane against motion
Friction is a boon because of its advantages like:

- Friction helps in walking

When we walk, we push the ground backwards with one foot. According to Newton’s Third law, there is an equal and opposite force exerted by the ground. This force is exerted on a comparatively smaller mass i.e. our foot. So, acceleration is increases. Hence the other foot gets accelerated.

If there is no friction, we will slip and can’t walk.

- Friction helps in movement of automobiles

Friction is a bane because of its disadvantages:

- A good amount of useful energy is wasted as heat in various machine parts
- Noise produced in machines
- Engines of automobiles consume more fuel

Methods to reduce Friction:

- Use of Lubricants




- Use of Grease
- Use of Ball bearing
  - Ball bearings are kind of rolling elements that use small freely rotating metal balls which reduce friction.
- Design modification of different parts of machine to reduce friction

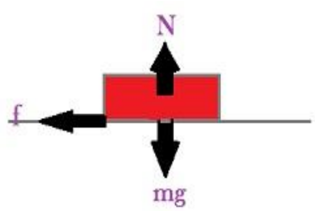
Circular motion

- Motion in a circular path
- Centripetal force plays a crucial role in circular motion.
- It is the Force that makes a body follow a curved path. It acts towards the centre.

- Generally denoted by F_c
- $F_c = mv^2 /r$

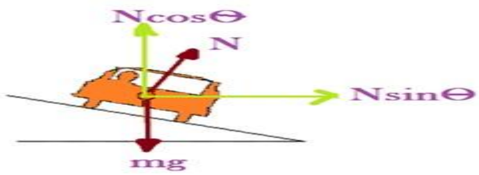
Sine we know that F = ma; therefore v^2 /r is termed as Centripetal acceleration

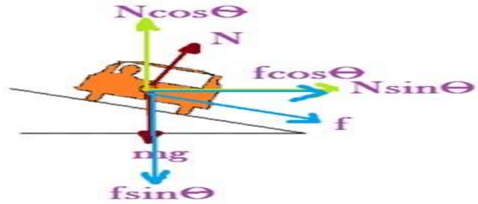
Motion of a car on a level road



N = mg
Static friction provides the centripetal acceleration
$f_s =< \mu_s N$
$mv^2/r =< \mu_s N$
$v^2 =< \mu_s NR/m = \mu_s mgR/m = \mu_s Rg$
$v =< \sqrt{\mu_s Rg}$
This is the maximum speed of a car in circular motion on a level road

Motion of a car on a banked road





In the vertical direction (Y axis)
$N\cos\Theta = f\sin\Theta + mg$ --------------------(i)
In horizontal direction (X axis)
$f\cos\Theta + N\sin\Theta = mv^2/r$ ----------------(ii)

Since we know that f $\mu_s N$
For maximum velocity, $f = \mu_s N$
(i)becomes:
$N\cos\Theta = \mu_s N\sin\Theta + mg$
Or, $N\cos\Theta - \mu_s N\sin\Theta = mg$
Or, $N = mg/(\cos\Theta - \mu_s\sin\Theta)$

Put the above value of N in (ii)
$\mu_s N\cos\Theta + N\sin\Theta = mv^2/r$
$\mu_s mg\cos\Theta/(\cos\Theta - \mu_s\sin\Theta) + mg\sin\Theta/(\cos\Theta - \mu_s\sin\Theta) = mv^2/r$
$mg(\sin\Theta + \mu_s\cos\Theta)/(\cos\Theta - \mu_s\sin\Theta) = mv^2/r$
Divide the Numerator & Denominator by cosϴ, we get
$v^2 = Rg(\tan\Theta + \mu_{s)})/(1 - \mu_s\tan\Theta)$
$\mathbf{v = \sqrt{} Rg(\tan\Theta + \mu_{s)})/(1 - \mu_s\tan\Theta)}$
This is the miximum speed of a car on a banked road.

Special case:
When the velocity of the car = v_0 ,
- No f is needed to provide the centripetal force. ($\mu_s = 0$)
- Little wear & tear of tyres take place.

$$\mathbf{v_o = \sqrt{} Rg(\tan\Theta)}$$

Problem: A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the
(a) optimum speed of the racecar to avoid wear and tear on its tyres, and
(b) maximum permissible speed to avoid slipping ?
Solution.
R = 300m
$\Theta = 15^o$
$\mu_s = 0.2$
- $v_o = \sqrt{} Rg\tan\Theta$
  $= \sqrt{300 * 9.8 * \tan 15^o}$
  = 28.1 m/s
- $v_{max} = \sqrt{} Rg(\tan\Theta + \mu_{s)})/(1 - \mu_s\tan\Theta)$
  $= \sqrt{} 300 * 9.8 * (0.2 + \tan 15^o)/(1 - 0.2\tan 15^o)$
  = 38.1 m/s

Thank You

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Notes provided by

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