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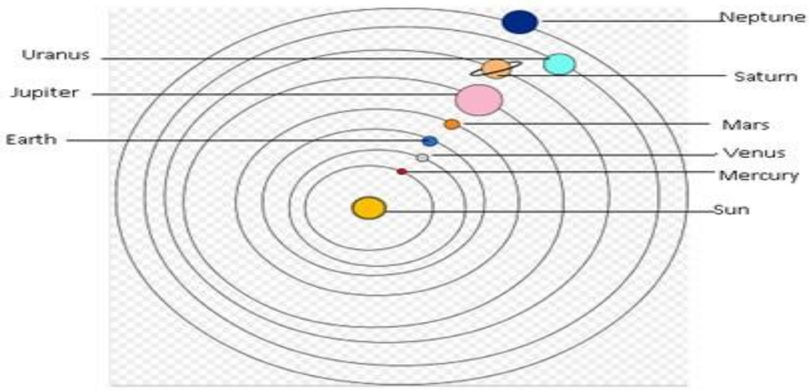
Introduction

- Whenever we throw an object towards the sky it will fall back onto the ground.
- For Example: - A ball comes down when thrown up. Rain drops fall towards the ground; Planets revolve in an elliptical orbit around sunetc.



Planets revolving in the elliptical orbit.

Rain drops falling on the earth.
Leaves fall off the tree.

- There is a force due to which all things are attracted towards the earth. This force is known as Gravitation.
- Gravitation is the force of attractionbetween all masses in the universe, especially the force of attraction exerted by the earth on all the bodies near its surface.
- In this chapter we will take a look at gravitation force, its laws, and we will also study about the planetary motion.

Gravitational Constant

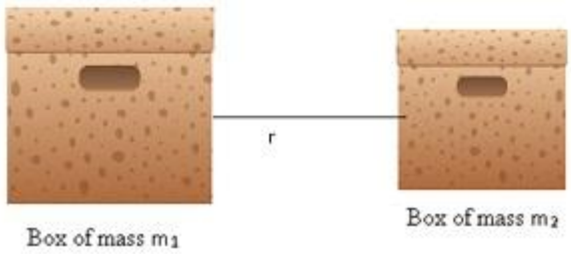
Newton's view of Gravitation

- Newtonwas the first scientist who studied the force of gravitation.
- According to him there is a force which is exerted by the surface of the earth because of which all the objects are attracted towards the surface of the earth.
- He also concluded that all objects in this universe attract each other with a force. This force is Gravitation force.

Universal Law of Gravitational

Universal law of Gravitation states –

- Everysingle body in this universe attracts each other with a force which is ∝to the product of their masses and inversely ∝to the square of the distance between them.
- This law holds good for all the bodies in the universe.
- If the product of mass of the bodies increase the force of attraction also increases between themand if the square of the distance between the bodies increases, force decreases.
- Mathematically:-
- Consider 2 boxes having mass m_1 and m_2. The distance between them is r.



- $F \propto m_1m_2$
- Force is ∝to the product of masses of 2 bodies.
- $F \propto 1/r^2$
- Force is inversely proportional to the square of the distance between the 2 bodies.
- Combining above equations:-
- $F \propto m_1m_2/r^2$
- **$F= G\ m_1m_2/r^2$**

Where G = universal Gravitational constant.

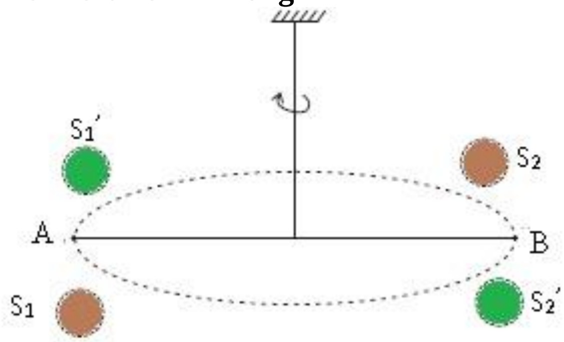
- Its value is constant and it never changes.
- m_1 and $m_2$are masses of 2 bodies.
- r = distance between the bodies.

Gravitational Constant: Cavendish experiment

- Cavendish performed an experiment to calculate the value of G.
- To calculate the value of G he took a wooden plank and attached two 2 balls on either side of the plank and hung this with a thin thread from the top.
- He introduced 2 very big balls and those balls are near the smaller balls.
- He observed that the small balls got attracted to big balls and wooden plank started rotating and as a result the thin thread started twisting.
- This happened because of force of attraction between the small balls and bigger balls.
- He observed that :-

1. Plank rotates till twisting force becomes equal to the gravitational force between the balls.
   $\tau = G\ m_1m_2/r^2$
   $L\theta = G\ m_1m_2/r^2$

- By using the above equation he calculated the value of G,
- **$G = 6.67 \times 10^{-11} Nm^2/kg^2$**
- Units of G : **Nm^2/kg^2**



Schematic drawing of Cavendish's experiment:-

- S_1 and S_2 are large spheresthat are kept on either side of the ellipse.
- Whenthe big spheres are taken to the other side of the ellipse (shown by dotted circles), the bar AB rotates a little since the torque reverses direction.
- The angle of rotation can be measured experimentally.

Problem:- Calculate the gravitational force of attraction between 2 lead balls of mass 20kg and 10kg separated by a distance of 10cm?
Answer: m_1= 20kg , m_2=10kg , r=10cm = 10/100 = 0.1m
$G=6.67 \times 10^{-11} Nm^2/kg^2$
$F=G\ m_1m_2/r^2$
$= 6.67 \times 10^{-11} \times 20 \times 10 / (0.1)^2$
$F = 1.3 \times 10^{-6} N$

Acceleration due to gravity of the earth

- Acceleration attained due to gravity of earth.
- All the objects fall towards the earth because of gravitational pull of the earth.
- And when abody is falling freely, it will have some velocity and therefore it will attain some acceleration. This acceleration is known as acceleration due to gravity.
- It is a vector quantity.
- Denoted by 'g'.
- Its value is $9.8 m/s^2$.

Example:-Stones falling from a rock will have some velocity because of which some acceleration. This acceleration is due to the force exerted by the earth on the rocks.This is known as acceleration due to gravity.

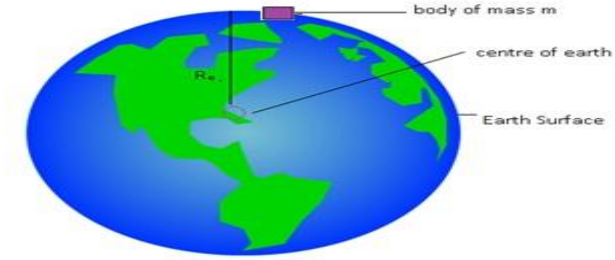
Stones falling from rock

Expression for Acceleration due to gravity

- Consider any object of mass 'm' at a point A on the surface of the earth.
- The force of gravity between the body and earth can be calculated as
- $F = G\, m\, M_e / R_e^2$ (1) where
  - m=mass of the body
  - M_e = mass of the Earth
  - R_e= distance between the body and the earthis same as the radius of the earth
- Newton's Second law states that
- F=ma (2)

Comparing the equations (1) and (2)

- $F = m\,(G\, m M_e / R_e^2)$
- $(G\, m M_e / R_e^2)$ is same as g(acceleration due to gravity)
- Therefore, the expression forAcceleration due to gravity.
- **$g = G\, M_e / R_e^2$**



Problem: Calculate the acceleration due to gravity on the (a) earth's surface (b) at a height of 1.5×10^5 m from the earth surface .Given: $R_e = 6.4\times10^6$m; $M_e = 5.98\times10^{24}$ kg?

Answer: -

(a) $g = GM_e/R_e^2$

$= 6.67\text{x}10\text{-}11\text{x}5.98\times10^{24}/(6.4\times10^6)^2$

$= 9.74 m/s^2$

(b) h= 1.5×10^5m (given)

$g(h) = GM_e/(R_e+h)^2$

$= 6.67\text{x}10\text{-}11\text{x}5.98\times10^{24}/ (6.4\times10^6 + 1.5\times10^5)$

$=9.30 m/s^2$

Problem:- Calculate the mass of the moon if the free fall acceleration near its surface is known to be $1.62 m/s^2$.Radius of the moon is 1738km?

Answer:

$g=1.62 m/s^2$

R_m = 1738km =1738 x 10^3 m

$G=6.67\times10^{-11} Nm^2/kg^2$

$g = GM_m/R_m^2$

$M_m = gR_m^2/G$

$=1.62\times(1738\times10^3)^2/6.67\times10^{-11}$

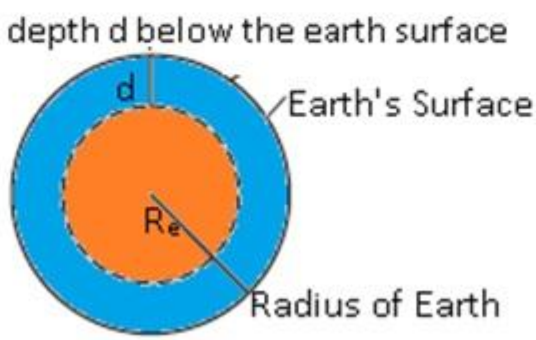
$=7.34\times10^{22}$ kg

Acceleration due to gravity below the surface of earth

- To calculate acceleration due to gravity below the surface of the earth (between the surface and centre of the earth).
- Density of the earth is constant throughout. Therefore,
- $\rho = M_e / (4/3\pi R_e^3)$ equation(1)
- where
- M_e = mass of the earth
- Volume of sphere= $4/3\pi R_e^3$
- R_e = radius of the earth.
- As entire mass is concentrated at the centre of the earth.
- Therefore density can be written as

$\rho = M_s / (4/3\pi R_s^3)$ equation (2)

- Comparing equation (1) and (2)
- $M_e/M_s = R_e^3/R_s^3$ where $R_s = (R_e\text{-}d)^3$
- d= distance of the body form the centre to the surface of the earth.
- Therefore,
- $M_e/M_s = Re^3/(R_e\text{-}d)^3$
- $M_s = M_e(R_e\text{-}d)^3/R_e^3$ from equation(3)
- To calculate Gravitational force (F) between earth and point mass m at a depth d below the surface of the earth.



Above figure shows the value of g at a depth d. In this case only the smaller sphere of radius (R_e–d) contributes to g.

- $F= G\, m\, M_s/(R_e\text{-}d)^2$
- g=F/m where g= acceleration due to gravity at point d below the surface of the earth.
- $g= GM_s/(R_e\text{-}d)^2$
- Putting the value of M_s from equation (3)

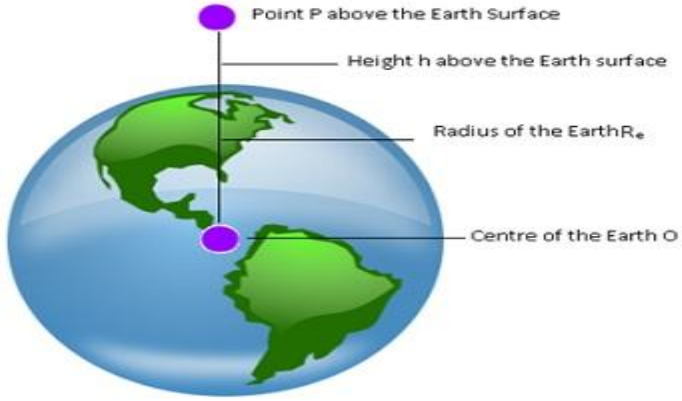
$= GM_e (R_e\text{-}d)^3/R_e^3(R_e\text{-}d)^2$

$=GM_e (R_e\text{-}d)/R_e^3$

- We know $g=GM_e/R_e^2$ equation (4)
- $g(d) = GM_e/ R_e^2 (1\text{-}d/R_e)$
- From equation (4)
- $\mathbf{g (d) = g(1\text{-}d/R_e)}$

Acceleration due to gravity above the surface of earth

- To calculate the value of acceleration due to gravity of a point mass m at a height h above thesurface of the earth.



Above figure shows the value ofacceleration due to gravity g at a height h above the surface of the earth.

- Force of gravitation between the object and the earth will be
- $F= G\ mM_e/(R_e+h)^2$ where
- m = mass of the object, R_e = radius of the earth
- $g(h) = F/m = GM_e/(R_e+h)^2 = GM_e/[R_e^2(1+h/R_e)^2]$
- $h << R_e$ (as radius of the earth is very large)
- By calculating we will get,

$g(h) = g(1-2h/R_e)$

- **Conclusion: -**The value of acceleration due to gravity varies on the surface, above the surface and below the surface of the earth.

Problem:- Assuming the earth to be a sphere of uniform mass density, how much would a body weigh half way down to the centre of the earth if it weighed 250 N on the surface?

Answer:

Weight of a body of mass m at the Earth's surface, W = mg = 250 N
Body of mass m is located at depth, $d=1/2\ R_e$
Where,
= Radius of the Earth
Acceleration due to gravity at depth g (d) is given by the relation:
$g' = (1-d/R_e)g$
$= (1- R_e/2x\ R_e)\ g= ½\ g$
Weight of the body depth d,
$W'=mg'$
$=m\ 1/2g =1/2mg=1/2W$
$=1/2\ x250=125N$

Problem:- A body weighs 63 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half the radius of the earth?

Answer: Weight of the body, W = 63 N
Acceleration due to gravity at height h from the Earth's surface is given by the relation:
$g'=g/(1+h/R_e)^2$
Where,
g = Acceleration due to gravity on the Earth's surface

R_e = Radius of the Earth
For h= R_e /2
$g' = g/(1+ R_e/2x\ R_e)^2 = g(1+1/2)^2 = 4/9g$
Weight of a body of mass m at height h is given as:
W'=mg'
=mx4/9g=4/9mg
=4/9W
=4/9x63=28N

Inertial and Gravitational Mass

Inertial Mass: - Inertial mass is defined as the mass of body by virtue of inertia of mass.

- By Newton's Law F=ma
- m=F/a where m= inertial mass (as it is because of inertia of a body)

Gravitational Mass:-Gravitational mass is defined as the mass of the body by virtue of the gravitational force exerted by the earth.

- By Gravitation Force of attraction –
- $F=GmM/r^2$
- $m=Fr^2/GM$ where
- m= mass of the object
- F=force of attraction exerted by the earth
- r=distance between object and earth
- M=mass of the earth
- Experimentally, Inertial mass= Gravitational mass

Problem:
Calculate the mass of the sun from the data given below:
Mean distance between Sun and Earth $=1.5x10^{11}m$
Time taken by earth to complete one orbit around the sun = 1 year
Answer:
$F= G\ M_e x M_s/r^2$
Given: $r= 1.5 \times 10^{11}m$, $v = 2\pi /r$; T= 1 year =365 days =365x24x60x60sec
$F_c= M_e v^2/r$
$F=F_c$
$GM_eM_s/r^2 = M_e v^2/r = M_e\ (2\pi\ r)^2 /rT^2$
After calculation:-
$M_s= 4n^2r^3/GT^2$
$M_s=2x10^{30}$ kg.

Problem: Let us assume that our galaxy consists of 2.5×10^{11} stars each of one solar mass. How long will a star at a distance of 50,000 ly from the galactic centre take to complete one revolution? Take the diameter of the Milky Way to be 10^5 ly.
Answer
Mass of our galaxy Milky Way, $M = 2.5 \times 10^{11}$ solar mass
Solar mass = Mass of Sun = 2.0×10^{36} kg
Mass of our galaxy, $M = 2.5 \times 1011 \times 2 \times 10^{36} = 5 \times 10^{41}$ kg
Diameter of Milky Way, $d = 10^5$ ly
Radius of Milky Way, $r = 5 \times 10^4$ ly
$1ly = 9.46 \times 10^{15}$ m
$r =5 \times 10^4 \times 9.46 \times 10^{15}$

$= 4.73 \times 10^{20}$ m
Since a star revolves around the galactic centre of the Milky Way, its time period is

given by the relation:

$T = (4\pi^2 r^3/G M)^{1/2}$
$= ((4 \times (3.14)^2 \times (4.73)^3 \times 10^{60})/6.67 \times 10^{-11} \times 5 \times 10^{41})^{1/2}$
$= (39.48 \times 105.82 \times 10^{30}/33.35)^{1/2}$
$= (125.27 \times 10^{30})^{1/2} = 1.12 \times 10^{16}$ s
1year = 365x324x60x60s

1s=1/365x324x60x60 years

Therefore,

$1.12 \times 10^{16}/365 \times 324 \times 60 \times 60$
$= 3.35 \times 10^{8}$ years.

Problem: - I_o, one of the satellites of Jupiter, has an orbital period of 1.769 days and the radius of the orbit is 4.22×10^{8} m. Show that the mass of Jupiter is about one-thousandth that of the sun.

Answer

Orbital period of $I_o = T_{Io}$ = 1.769 days =1.769x24x60x60s
Orbital radius of $I_o = R_{Io} = 4.22 \times 10^{8}$ m
Satellite I_o is revolving around the Jupiter
Mass of the latter is given by the relation:

$M_j = 4\pi^2 R_{Io}^3/G T_{Io}^2$
Where,

M_j = Mass of Jupiter
G = Universal gravitational constant

Orbital radius of the Earth,

T_e = 365.25 days = 365.25x24x60x60s
Orbital radius of the Earth,

$R_e = 1AU = 1.496 \times 10^{11}$m
Mass of the Sun is given as:

$M_s = 4\pi^2 R_e^3/G T_e^2$
$M_s/M_j = (4\pi^2 R_e^3/G T_e^2) \times (G T_{Io}^2/4\pi^2 R_{Io}^3)$
$= (R_e^3/R_{Io}^3) \times (T_{Io}^2/T_e^2)$
$= (1.769 \times 24 \times 60 \times 60s/365.25 \times 24 \times 60 \times 60s)^2 \times (1.496 \times 10^{11}m/4.22 \times 10^{8}$ m)
=1045.04

$M_s/M_j \sim 1000$
$M_s \sim 1000 M_j$
Hence, it can be inferred that the mass of Jupiter is about one-thousandth that of the Sun.

Problem: - How will you 'weigh the sun', that is estimate its mass? The mean orbital radius of the earth around the sun is 1.5×10^8 km.

Answer:

Orbital radius of the Earth around the Sun, $r = 1.5 \times 10^{11}$ m

Time taken by the Earth to complete one revolution around the Sun,

T = 1 year = 365.25 days= 365.25 × 24 × 60 × 60 s

Universal gravitational constant, $G = 6.67 \times 10^{-11}$ $Nm^2\ kg^{-2}$

Thus, mass of the Sun can be calculated using the relation

$M=4\ \pi^2 r^3/GT^2$

$= (4x\ (3.14)^2 x(1.5x10^{11})^3)/(6.67x10^{-11}\ x(365.25 \times 24 \times 60 \times 60))$

$=133.24x10/6.64x10^4$

$=2.0x10^{30}Kg$

Hence, the mass of the Sun is 2 × 1030 kg.

Gravitational Potential Energy

- Potential energy is due to the virtue of position of the object.
- Gravitational Potential Energy is due to the potential energy of a body arising out of the force of gravity.
- Consider a particle which is at a point P above the surface of earth and when it falls on the surface of earth at position Q, the particle is changing its position because of force of gravity.
- The change in potential energy from position P to Q is same as the work done by the gravity.
- It depends on the height above the ground and mass of the body.

Stationary roller-coaster

Expression for Gravitational Potential Energy

Case1:- 'g' is constant.

- Consider an object of mass 'm' at point A on the surface of earth.
- Work done will be given as :
- W_{BA}=F X displacement where F = gravitational force exerted towards the earth)
- $=mg(h_2-h_1)$ (body is brought from position A to B)
- $=mgh_2-mgh_1$
- $\mathbf{W_{AB}=V_A-V_B}$
- where
  - V_A=potential energy at point A
  - V_B= potential energy at point B

- From above equation we can say that the work done in moving the particle is just the difference of potential energy between its final and initial positions.

Case2:-'g' is not constant.

- Calculate Work done in lifting a particle from $r = r_1$ to $r = r_2$ ($r_2 > r_1$) along a vertical path,
- We will get , $W=V(r_2) - V(r_1)$

Conclusion: -

- In general the gravitational potential energy at a distance 'r' is given by :

$$V(r) = -GM_e m/r + V_o$$

- where
- V(r) = potential energy at distance 'r'
- V_o = At this point gravitational potential energy is zero.
- Gravitational potential energy is ∝ to the mass of the particle.

Gravitational Potential

- Gravitational Potential is defined as the potential energy of a particle of unit mass at that point due to the gravitational force exerted byearth.
- Gravitational potential energy of a unit mass is known as gravitational potential.
- Mathematically:
- $G_{potential} = -GM/R$

Problem:

Choose the correct alternative:

Acceleration due to gravity increases/decreases with increasing altitude.

Acceleration due to gravity increases/decreases with increasing depth. (assume the earth to be a sphere of uniform density).

Acceleration due to gravity is independent of mass of the earth/mass of the body.

The formula $-G\, M_m (1/r_2 - 1/r_1)$ is more/less accurate than the formula $mg(r_2 - r_1)$ for the difference of potential energy between two points r_2 and r_1 distance away from the centre of the earth.

Answer:

(a)Decreases

(b)Decreases

(c)Mass of the body

(d)More

Explanation:

- Acceleration due to gravity at depth h is given by the relation:

$g_h = (1- 2h/R_e)g$

Where,

R_e = Radius of the Earth, g = acceleration due to gravity on the surface of the earth.

It is clear from the given relation that acceleration due to gravity decreases with an increase in height.

- Acceleration due to gravity at depth d is given by the relation:

$g_d=(1-d/R_e)g$

It is clear from the given relation that acceleration due to gravity decreases with an increase in depth.

- Acceleration due to gravity of body of mass m is given by the relation: $g=GM/r^2$

Where,

G = Universal gravitational constant
M = Mass of the Earth
R = Radius of the Earth
Hence, it can be inferred that acceleration due to gravity is independent of the mass of the body.

- Gravitational potential energy of two points r_2 and r_1 distance away from the centre of the Earth is respectively given by:

$V(r_1) = -G\,mM/r_1$
$V(r_2) = -G\,mM/r_2$
Therefore,
Difference in potential energy, $V = V(r_2) - V(r_1) = -GmM\,(1/r_2 - 1/r_1)$
Hence, this formula is more accurate than the formula $mg\,(r_2 - r_1)$.

Problem:-
Two earth satellites A and B each of mass m are to be launched into circular orbits earth's surface at altitudes 6400km and 1.92×10^4 km resp. The radius of the Earth is 6400km.Find (a) The ratio of their potential energies and (b) the ratio of their kinetic energies. Which one has greater total energy?
Answer:-

- m_a = mass of satellite A

m_b=mass of satellite B
h_a=6400km, $h_b=1.92\times10^4$ km
R_e=6400km
Potential Energy = $-GM_em/(R_e+h)$
For A $(P.E)_A = -GM_em/(6400+6400)$
$=-GM_em/12800$ ---(1)
For B$(P.E)_B = -GM_em/(6400+1.92\times10^4)$
$-GM_em/(6400+19200)$ ---(2)
Divide 1 by 2 we will get

$(P.E)_A/(P.E)_B = 2 : 1$

- $(K.E)_A = GMm/2\times12800$ (3)

$(K.E)_B = GMm/2(1.92\times10^4+6400)$ (4)
Dividing (3) by (4)

$(K.E)_A/(K.E)_B = GMm/(12800) \times 2(1.92\times10^4+6400)/GMm$
$(K.E)_A/(K.E)_B = 2:1$

- Total Energy of A = $-GMm/2r$ r=12800km

Total Energy of B = $-GMm/2r$ $r=(1.92\times10^4+6400)$km
Total energy of B is greater than A.

Problem: - A rocket is fired vertically with a speed of 5 km s^{-1} from the earth's surface. How far from the earth does the rocket go before returning to the earth? Mass of the earth = 6.0×10^{24} kg; mean radius of the earth = 6.4×10^6m; G= 6.67×10^{-11} N m^2 kg^{-2}.

Answer: Velocity of the rocket, $v = 5$ km/s $= 5\times10^3$ m/s
Mass of the Earth, $M_e = 6.0\times10^{24}$ kg
Radius of the Earth, $R_e=6.4\times10^6$m
Height reached by rocket mass, m = h

At the surface of the Earth,

Total energy of the rocket = Kinetic energy + Potential energy

$= 1/2mv^2+(-GM_em/R_e)$
At highest point h,

v=0

And Potential Energy = $-(GM_em/R_e+h)$
Total energy of the rocket

$=0+ - (GM_em/R_e+h)$
$=-(GM_em/R_e+h)$
Total energy of the rocket

From the law of conservation of energy, we have

Total energy of the rocket at the Earth's surface = Total energy at height h.

$1/2mv^2+(-GM_em/R_e) = - GM_em/R_e+h$
$1/2v^2 = GM_e(1/ R_e - 1/ R_e+h)$
By calculating

$1/2v^2 = gR_eh/R_e+h$
Where $g=GM/R_e^2 = 9.8m/s^2$
Therefore,

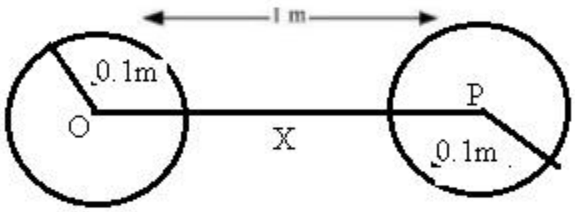
$v^2(R_e+h) = (2gR_eH)$
$v^2R_e=h (2gR_e-v^2)$
$h=R_e-v^2/ (2gR_e-v^2)$
$=6.4 \times 10^6x(5x10^3)^2/2x9.8x6.4x10^6-(5x10^3)^2$
$h=1.6x10^6m$
Height achieved by the rocket with respect to the centre of the Earth

$=R_e+h$
$=6.4 \times 10^6x1.6x10^6$
$=8.0x10^6m$
The distance of the rocket is 8×10^6 m from the centre of the Earth.

Problem: - Two heavy spheres each of mass 100 kg and radius 0.10 m are placed 1.0 m apart on a horizontal table. What is the gravitational force and potential at the mid-point of the line joining the centres of the spheres? Is an object placed at that point in equilibrium? If so, is the equilibrium stable or unstable?
Answer:

- 0;
- -2.7×10^{-8} J /kg;
- Yes;
- Unstable

Explanation:-
The situation is represented in the given figure:



Mass of each sphere, M = 100 kg
Separation between the spheres, r = 1m
X is the mid-point between the spheres. Gravitational force at point X will be zero. This is because gravitational force exerted by each sphere will act in opposite directions.
Gravitational potential at point X:
=-GM/(r/2) – GM(r/2) = -4GM/r
$=4x6.67x10^{-11}x100$
$=-2.67x10^{-8}$J/kg
Any object placed at point X will be in equilibrium state, but the equilibrium is unstable. This is because any change in the position of the object will change the effective force in that direction.

Problem:-
Two stars each of one solar mass (= 2× 10^{30} kg) are approaching each other for a head on collision. When they are a distance 10^9 km, their speeds are negligible. What is the speed with which they collide? The radius of each star is 10^4 km. Assume the stars to remain undistorted until they collide. (Use the known value of G).
Answer:-
Mass of each star, M = 2 × 10^{30} kg
Radius of each star, R = 10^4 km = 10^7 m
Distance between the stars, r = 10^9 km = 10^{12}m
For negligible speeds, v = 0 total energy of two stars separated at distance r

$=-GMM/r + 1/2mv^2$
=-GMM/r + 0 (i)

Now, consider the case when the stars are about to collide:

Velocity of the stars = v

Distance between the centres of the stars = 2R

Total kinetic energy of both stars = $1/2Mv^2 + 1/2Mv^2 = Mv^2$
Total potential energy of both stars =-GMM/2R

Total energy of the two stars $=Mv^2$ - GMM/2R (ii)
Using the law of conservation of energy, we can write:

Mv^2 - GMM/2R =- GMM/r
v^2=-GM /r + GM/2R = GM (-1/r + 1/2R)
$=6.67x10^{-11}$ x 2 $x10^{30}(-1/10^{12} + 1/2x10^7)$
$=13.34x10^{19}(-10^{-12}+5x10^{-8})$
$=13.34x10^{19}x 5 x 10^{-8}$
$=6.67x10^{12}$
$v=\sqrt{6.67x10^{12}}= 2.58x10^6$m/s

Problem:- A 400kg satellite is in circular orbit of radius $2R_E$ about the Earth. How much energy is required to transfer it to circular orbit of radius $4R_E$?What are the changes in the kinetic and potential energies?

Answer:

$E_i = - GM_e m/2R_E$

$E_f = - GM_e m/4R_E$

$\Delta E = E_f - E_i$

$= - GM_e m/2R_E(1/4-1/2)$

$\Delta E = GM_e m/8 R_E$

In terms of 'g'

$\Delta E = gm R_E/8$

By putting the values and calculating

$\Delta E = 3.13 \times 10^9 J$

The energy which is required to transfer the satellite to circular orbit of radius $4R_E$ is $3.13 \times 10^9 J$.

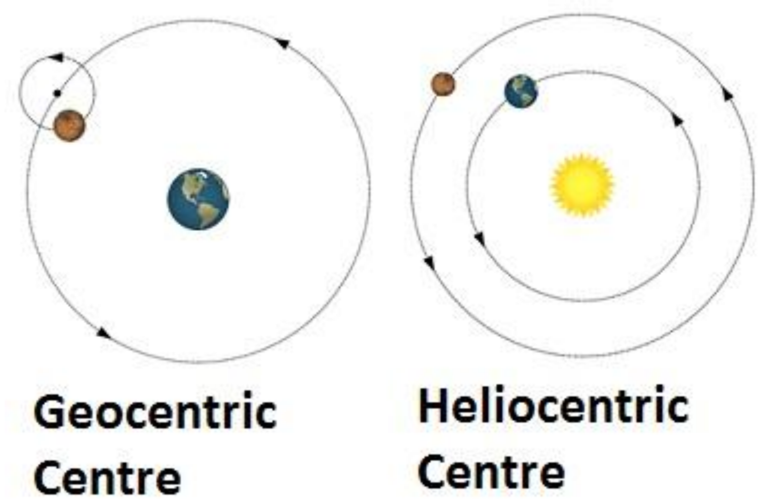
Change in Kinetic energy $\Delta k = k_f - k_i$

$\Delta k = 3.13 \times 10^9 J$

Change in Potential energy ΔV= 2xΔE = -6.25x109J

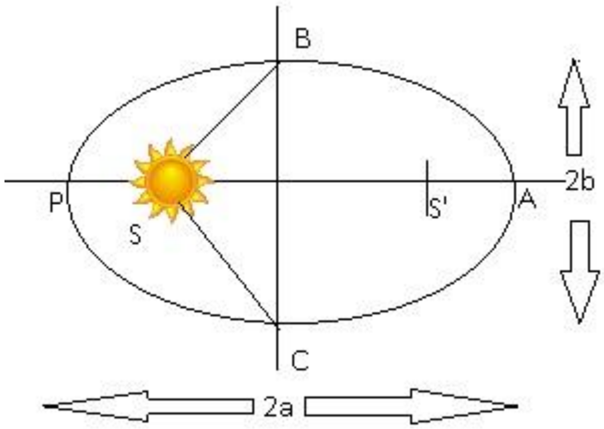
Planetary Motion

- Ptolemy was the first scientist who studied the planetary motion.
- He gave geocentric model. It means all the planets, stars and sun revolve around the earth and earth is at the centre.
- Heliocentric model was proposed by some Indian astronomers.
- According to which all planets revolve around the sun.
- Nicholas proposed the Nicholas Copernicus modelaccording to which all planets move in circles around the sun.
- After Nicholas one more scientist named Tycho Brahe did lot of observations on planets.
- Finally came Johannes Kepler who used Tycho Brahe observations and he gave Kepler's 3 laws of Gravitation.
- These 3 laws became the basis of Newton's Universal law of Gravitation.



Kepler's 1st Law: Law of Orbits

Statement: - The orbit of every planet is an ellipse around the sun with sun at one of the two foci of ellipse.



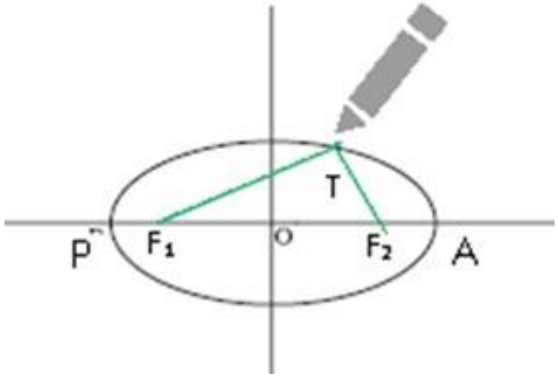
Whenever a planet revolves around sun it traces an ellipse around the sun. The closest point is P and thefarthest point is A, P is called the perihelion and A the aphelion. The semi major axis is half the distance AP.

Kepler's 1st law Vs. Copernicus Model

- According to Copernicus planets move in circular motion whereas according to Kepler planets revolve in elliptical orbit around the sun.
- Copernicus model is based on one special case because circle is a special case of ellipse whereas Kepler's laws aremore of ageneral form.
- Kepler's law also tells us about the orbits which planets follow.

To Show ellipse is a special form of Circle

- Select two points F1 and F2.
- Take a pieceof string and fix its ends at F_1 and F_2.
- Stretch the string taut with the help of a pencil and then draw a curve by moving the pencil keeping the string taut throughout. Fig. (a).
- The resulting closed curve is an ellipse. For any point T on the ellipse, the sum of distances from F_1 and F_2 is a constant. F_1,F_2 are called the foci.
- Join the points F_1 and F_2,and extend the line to intersect the ellipse at points P and A as shown in Fig. (a).
- The centre point of the line PA is the centre of the ellipse O and the length PO = AO, which is also known as the semimajor axis of the ellipse.
- For a circle, the two foci merge onto one and the semi-major axis becomes the radius of the circle.



Fig(a)

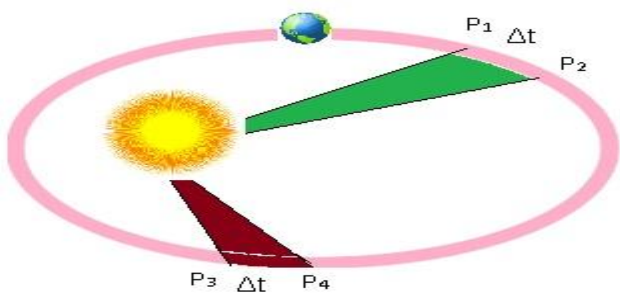
A string has its ends fixed at F1 and F2. The tip of the pencil holdsthe string taut and is moved around and we will get an ellipse.

Kepler's 2^{nd} law: Law of Areas

Statement:-The line that joins a planet to the sun sweeps out equal areas in equal intervals of time.

- Area covered by the planet while revolving around the sun will be equal in equal intervals of time. This means the rate of change of area with time is constant.
- Suppose position and momentum of planet is denoted by 'r' and 'p' and the time taken will be Δt.
- $\Delta A = 1/2 x r x v \Delta t$ (where $v\Delta t$ is distance travelled by a planet in Δt time.)
- $\Delta A / \Delta t = 1/2(\mathbf{r} \times \mathbf{v})$
- where
- (Linear momentum) p=mv or we can write as
- $\mathbf{v} = \mathbf{p}/m$
- $=1/2m(\mathbf{r} \times \mathbf{p})$
- $=1/2\ L/2m$ where L= angular momentum(It is constant for any central force)

$\Delta A / \Delta t$ = constant (This means equal areas are covered in equal intervals of time).



Kepler's 3^{rd}Law: Law of periods

Statement: -

- According to this law the square of time period of a planet is $\propto$ to the cube of the semi-major axisof its orbit.
- Suppose earth is revolving around the sun then the square of the time period (time taken to complete one revolution around sun) is $\propto$ to the cube of the semi major axis.
- It is known as Law of Periods as it is dependent on the time period of planets.
- Derivation of 3^{rd} Law: assumption: The path of the planet is circular.
- Let m=mass of planet
- M= mass of sun

- According to Newton's Law of Gravitation:
- $F= GMm/r^2$
- $F_c=mv^2/r$
- where
- F_c =centripetal force which helps the planet to move around sun in elliptical order.
- F = Fc
- $GMm/r^2= mv^2/r$ where r=radius of the circle
- $GM/r= v^2$ (1)
- $v= 2\pi r/T$
- Squaring both the sides the above equation
- $v^2=4\pi^2 r^2/T^2$
- putting the value (1)
- $GM/r=4\pi r^2/T^2$
- $T^2= (4\pi^2 r^3/GM)$ where $(4\pi^2/GM)$ = constant
- $\mathbf{T^2=r^3}$ (In ellipse semi-major axis is same as radius of the circle)

Problem: - Suppose there existed a planet that went around the sun twice as fast as the earth.What would be its orbital size as compared to that of the earth?
Answer: Lesser by a factor of 0.63
Time taken by the Earth to complete one revolution around the Sun,

T_e = 1 year
Orbital radius of the Earth in its orbit, R_e = 1 AU
Time taken by the planet to complete one revolution around the Sun,

$T_p=1/2T_e=1/2$ year
Orbital radius of the planet = R_p
From Kepler's third law of planetary motion, we can write:

$(R_p/R_e)^3 = (T_p/T_e)^2$
$R_p/R_e = (T_p/T_e)^{2/3}$
$=((1/2)/1)^{2/3} =(0.5)^{2/3} = 0.63$
Hence, the orbital radius of the planet will be 0.63 times smaller than that of the Earth.

Problem: A Saturn year is 29.5 times the earth year. How far is the Saturn from the sun, if theearth is 1.50×10^8 km away from the sun?

Answer: Distance of the Earth from the Sun, $R_e = 1.5 \times 10^8$ km $= 1.5 \times 10^{11}$ m
Time period of the Earth = T_e
Time period of Saturn, $T_s = 29.5\ T_e$
Distance of Saturn from the Sun = R_s
From Kepler's third law of planetary motion, we have

$T^2= (4\pi^2 r^3/GM)^{1/2}$
For Saturn and Sun, we can write

$(R_s^3/ R_e^3) = T_s^2/ T_e^2$
$R_s = R_e(T_s/ T_e)^{2/3}$
$=1.5 \times 10^{11}(29.5)^{2/3}$
$=1.5 \times 10^{11}x9.55$
$=14.32x10^{11}$m
Hence, the distance between Saturn and the Sun is $1.432x10^{12}$m.

Problem:-Let the speed of the planet at the perihelion P in Fig. be v_P and Sun-planet distance SP is r_P. Relate $\{r_P, v_P\}$ to the corresponding quantities at aphelion $\{r_A, v_A\}$. Will the planet take equal times to traverse BAC and CPB?

Answer The magnitude of the angular momentum at P is Lp = mp rp vp, since inspection tells us that r_p and v_p are mutually perpendicular. Similarly,
$L_A = m_p r_A v_A$. Fromangular momentum conservation
$m_p r_p v_p = m_p r_A v_A$
or$v_p/v_A=r_a/r_p$
Since $r_A > r_p$, $v_p > v_A$.
The area SBAC bounded by the ellipse andthe radius vectors SB and SC is larger than SBPCin Fig. From Kepler's second law, equal areasare swept in equal times. Hence the planet willtake a longer time to traverse BAC than CPB.

Escape Velocity

- Escape velocity is the minimum velocity that a body must attain to escape the gravitational field of the earth.
- Suppose if we throw a ball,it will fall back. This is happening due to the force of gravitation exerted on the ball by the surface of the earth due to which the ball is attracted towards the surface of the earth.
- If we increase the velocity to such an extent that the object which is thrown up will never fall back.This velocity is known as escape velocity.

Ball is thrown up but it falls down because of force of gravitation.

The same ball is thrown with a velocity that itescapes the force of gravitation of earth and does not come back. This velocity is known as escape velocity.

- Mathematically:-
- Suppose we throw a ball and the initial velocity of the ball is equal to the escape velocity such that ball never comes back.
- Final Position will be infinity.
- **At Final Position**: At Infinity
- Total Energy (∞) = kinetic Energy (∞) + PotentialEnergy (∞)
- KineticEnergy (∞) = ½ mv_f^2 where v_f=final velocity
- Potential Energy (∞) = -GMm/r + V_0
- where M=mass of the earth, m= mass of the ball,

V_0=potential energy at surface of earth, r=∞ r=distance from the centre of the earth.

- Therefore: - Potential Energy (∞) =0

- Total Energy (∞) $=\frac{1}{2} mv_f^2$ (1)
- **At initial position**:-
- E. $= 1/2mv_i^2$
- $E= -GMm/ (R_e+h) + V_0$
- Where h= height of the ball from the surface of the earth.
- Total Energy (initial) $= 1/2mv_i^2 - GMm/ (R_e+h)$ (2)
- According to law of conservation of energy
- Total Energy (∞) =Total Energy (initial)
- $\frac{1}{2} mv_f^2 = 1/2mv_i^2 - GMm/ (R_e+h)$
- As L.H.S = positive
- $1/2mv_i^2 - GMm/ (R_e+h) \geq 0$
- $1/2mv_i^2 = GMm/ (R_e+h)$
- By calculating
- $v_i^2 = 2GM/ (R_e+h)$
- Assume Ball is thrown from earth surface $h<<R_e$
- This implies R_e+h is same as Re as we can neglect h.
- Therefore,$v_i^2 = 2GM/ (R_e)$
- Or $\mathbf{v_i = \sqrt{(2GM/R_e)}}$
- This is the initial velocity with which if the ball is thrown it will never fall back on the earth surface.

In terms of 'g'

- $g = GM/R_e^2$
- Escape velocity can be written as

$\mathbf{V_e= \sqrt{2gR_e}}$

Example of Escape Velocity: No atmosphere on moon

- Earth Escape velocity= $V_e= \sqrt{2gR_e}$
- Moon Escape velocity = $V_e= \sqrt{2g_mR_m}$ where R_m is the radius of the moon.
- $g_m=1/6\ g_e$; $R_m = 1/4\ R_e$
- $(V_e)_{moon}= \sqrt{2\ g_mR_m} = \sqrt{2xg/6x\ R_e/4}$
- After calculating we will get:

$\mathbf{(V_e)_{moon} = 1/5\ (V_e)_{earth} = 2.3km/s}$

- As this velocity is very less, the molecules cannot accumulate on the moon so there is no atmosphere on the moon.

Problem: - Does the escape speed of a body from the earth depend on
(a) the mass of the body,
(b) the location from where it is projected,
(c) the direction of projection,
(d) the height of the location from where the body is launched?

Answer

(a) No
(b) No
(c) No
(d) Yes

Escape velocity of a body from the Earth is given by the relation:
$V_e= \sqrt{2gR_e}$
g = Acceleration due to gravity

R = Radius of the Earth
It is clear from equation (i) that escape velocity is independent of the mass of the body and the direction of its projection. However, it depends on gravitational potential at the point from where the body is launched. Since this potential marginally depends on the height of the point, escape velocity also marginally depends on these factors.

Problem: A comet orbits the Sun in a highly elliptical orbit. Does the comet have a constant (a) linear speed, (b) angular speed, (c) angular momentum, (d) kinetic energy, (e) potential energy, (f) total energy throughout its orbit? Neglect any mass loss of the comet when it comes very close to the Sun.

Answer:-

(a) No
(b) No
(c) Yes
(d) No
(e) No
(f) Yes

Angular momentum and total energy at all points of the orbit of a comet moving in a highly elliptical orbit around the Sun are constant. Its linear speed, angular speed, kinetic and potential energy varies from point to point in the orbit.

Problem: - The escape speed of a projectile on the earth's surface is 11.2 km s^{-1}. A body is projected out with thrice this speed. What is the speed of the body far away from the earth? Ignore the presence of the sun and other planets.

Answer:-

Escape velocity of a projectile from the Earth, v_{esc} = 11.2 km/s
Projection velocity of the projectile, vp = $3v_{esc}$
Mass of the projectile = m

Velocity of the projectile far away from the Earth = v_f
Total energy of the projectile on the Earth=$1/2m_p^2 – 1/2mv_{esc}^2$
Gravitational potential energy of the projectile far away from the Earth is zero.

Total energy of the projectile far away from the Earth =$1/2mv_f^2$
From the law of conservation of energy, we have,

$1/2m_p^2 – 1/2mv_{esc}^2 = 1/2mv_f^2$
$v_f = \sqrt{v_p^2 - v_{esc}^2}$
$= \sqrt{(3\ v_{esc})^2 - (v_{esc})^2}$
$= \sqrt{8}\ v_{esc}$
=31.68km/s

Problem:-

Calculate the escape velocity on the surface of the moon? Given that the radius of the moon is 1.7×10^6m and the mass of the moon is 10^{22}kg.

Answer:-

$V_e = \sqrt{2GM/R_e}$
$R_m = 1.7 \times 10^6$m
M= 10^{22}kg
G=$6.67 \times 10^{-11} Nm^2/kg^2$
$V_e = \sqrt{2 \times 6.67 \times 10^{-11} \times 10^{22} / 1.7 \times 10^6}$
=2.4×10^3m/s
The escape velocity on the surface of the moon is 2.4×10^3m/s.

Problem:- What is the escape velocity from Jupiter given that the mass is 300 times that of the Earth's and its radius is 10times larger?

Answer:-

$M_J = 300M_e$

$R_J = 10R_e$

$V_e = \sqrt{2GM_e/R_e} = V_e = \sqrt{2GM_J/R_J}$

$= \sqrt{2G \times 300M_e/10 \times R_e}$

After putting the values,

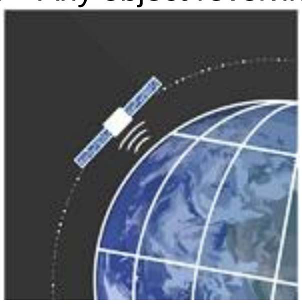
$= \sqrt{30} \times 11.2$m/s

=61.3km/s

The escape velocity on the surface of the Jupiter is 61.3km/s.

Earth Satellites

- Any object revolving around the earth.



Natural Satellite

- Satellitecreated by nature.
- Example: - Moon is the only natural satellite of earth.

Artificial Satellites:

- Human built objects orbiting the earth for practical uses. There are several purposes which these satellites serve.
- Example:- Practical Uses of Artificial satellites
- Communication
- Television broadcasts
- Weather observation
- Military support
- Navigation
- Scientific research

Determining the Time Period of Earth Satellite

- Time taken by the satellite to complete one rotation around the earth.
- As satellites move in circular orbits there will be centripetal force acting on it.
- $F_c=mv^2/R_e+h$ It is towards the centre.
- Where
  - h= distance of satellite form the earth
  - F_c= centripetal force
- $F_G= GmM_e/(R_e+h)^2$
- where
  - F_g= Gravitation force
  - m= mass of the satellite
  - M_e = mass of the earth
- $F_c=F_G$
- $mv^2/R_e+h = GmM_e/(R_e+h)^2$
- $v^2=GM_e/R_e+h$
- $\mathbf{v=\sqrt{} GM_e/R_e+h}$ (1)
- This is the velocity with which satellite revolve around the earth.
- The satellite covers distance = 2 $\pi(R_e+h)$ with velocity v.
- $T=2\pi(R_e+h)/v$
- $2\pi(R_e+h)/\sqrt{}GM_e/R_e+h$ From (1)
- $\mathbf{T=2\pi(R_e+h)^{3/2}/\sqrt{}GM_e)}$

Special Case:-

1. h<< R_e (satellite is very near to the surface of the earth)
   - Then $T=2\pi\sqrt{} R_e^3/GM_e$
   - After calculating

T=2 $\pi\sqrt{}$ $\mathbf{R_e/g}$

Energy of an orbiting satellite

- m= mass of the satellite, v=velocity of the satellite
- E.=$1/2mv^2$
- =1/2 m (GM_e/R_e+h) by using (1)
- **E. =1/2 $\mathbf{GM_e/(R_e+h)}$**
- **E.= $\mathbf{-GM_em/(R_e+h)}$**

- Total Energy = K.E. + P.E.
- $=1/2\ GM_e/(R_e+h) + -GM_em/(R_e+h)$
- **E.= $GM_em/2(R_e+h)$**
- Conclusion:-
- P.E. = 2 x K.E.
- Total energy is negative. This means the satellite cannot escape from the earth's gravity.

Problem: - Choose the correct alternative:
If the zero of potential energy is at infinity, the total energy of an orbiting satellite is negative of its kinetic/potential energy. The energy required to launch an orbiting satellite out of earth's gravitational influence is more/less than the energy required to project a stationary object at the same height (as the satellite) out of earth's influence.
Answer:
(a)Kinetic energy
(b)Less
(a)Total mechanical energy of a satellite is the sum of its kinetic energy (always positive) and potential energy (may be negative). At infinity, the gravitational potential energy of the satellite is zero. As the Earth-satellite system is a bound system, the total energy of the satellite is negative.
Thus, the total energy of an orbiting satellite at infinity is equal to the negative of its Kinetic Energy.
(b)An orbiting satellite acquires a certain amount of energy that enables it to revolve around the Earth. This energy is provided by its orbit. It requires relatively lesser energy to move out of the influence of the Earth's gravitational field than a stationary object on the Earth's surface that initially contains no energy.
Problem:
The planet Mars has two moonsPhobos and delmos. (i) Phobos has a period 7 hours, 39 minutes and an orbital radius of 9.4 × 103 km. Calculate the massof mars. (ii) Assume that earth and mars move in circular orbits around the sun,with the Martian orbit being 1.52 times the orbital radius of the earth. What isthe length of the Martian year in days?

Answer:
By using $T^2 = k\ (R_E + h)^3$ where $(k=4\pi^2 / GM_E)$
$T^2= (4\pi^2 R^3/GM_m)$
$M_m= 4\pi^2 R^3/ G\ T^2$
$=4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}/ (6.67\times10^{-11} \times (459\times60)^2)$
$=4 \times (3.14)^2 \times (9.4)^3 \times 10^{18}/ (6.67 \times (4.59\times6)2\times10^{-5})$
$=6.48\times10^{23}$kg
(ii) Once again By Kepler's third law

$T_m^2/T_e^2 = R^3_{Ms}/R^3_{ES}$
Where R_{MS} is the mars -sun distance and R_{ES} is the earth-sun distance.
$\therefore T_M = (1.52^{)3/2} \times 365$
= 684 days

We note that the orbits of all planets except Mercury, Mars and Pluto are very close to being circular. For example, the ratio of the semi minor to semi-major axis for our Earth is, b/a = 0.99986.

Problem:- Weighing the Earth: You are given the following data: $g = 9.81\ ms^{-2}$, $R_E = 6.37\times 10^6$ m, the distance to the moon $R = 3.84\times 10^8$ m and the time period of the moon's revolution is 27.3 days. Obtain the mass of the Earth M_E in two different ways.

Answer:

$M_E = g\ R_E^2/G$

$=9.81 \times (6.37\times 10^6)^2/6.67\times 10^{-11}$

$= 5.97\times 10^{24}$ kg.

The moon is a satellite of the Earth. From the derivation of Kepler's third law

$=4\ \pi^2\ R^3/GM_E$

$M_E=4\ \pi^2\ R^3/GT^2$

$=(4\text{x}3.14\text{x}3.14\text{x}(3.84)^3\text{x}1024)/6.67\text{x}10^{-11\text{x}}(27.3\text{x}24\text{x}60\text{x}60)^2$

$= 6.02 \times 10^{24}$ kg

Both methods yield almost the same answer, the difference between them being less than 1%.

Problem:- Express the constant k of in Eq. $T^2 = k\ (R_E + h)^3$ where ($k=4\ \pi^2\ /\ GM_E$) in days and kilometres. Given $k = 10^{-13}\ s^2\ m^{-3}$. The moon is at a distance of 3.84×10^5 km from the earth. Obtain its time-period of revolution in days.

Answer Given

$k = 10^{-13}\ s^2\ m^{-3}$

$=10^{-13}[d^2/\ (24\text{x}60\text{x}60)^2]\ [1/\ (1/1000)^3 km^3]$

$= 1.33 \times 10^{-14}\ d^2\ km^{-3}$

Using Eq. $T^2 = k\ (R_E + h)^3$

- and the given value of k, where ($k=4\ \pi^2\ /\ GM_E$) the time period of the moon is

$T^2 = (1.33 \times 10^{-14})\ (3.84 \times 10^5)^3$

$T = 27.3$ d

Using Eq. $T^2 = k\ (R_E + h)^3$ also holds for elliptical orbits if we replace (R_E+h) by the semi-major axis of the ellipse. The earth will then be at one of the foci of this ellipse.

Geostationary Satellite:-

- Geo means earth and stationary means at rest. This means something which is stationary.
- Satellites orbiting around the Earth in equatorial plane with time period equal to 24 hours.

- Appear to be stationary with respect to earth. They also rotate around earth with time period of 24 hours.
- These satellites can receive telecommunication signals and broadcast them back to a wide area on earth.
- Example: INSAT group of satellites.

Problem:- Calculate the height of a geostationary satellite from the surface of the earth?
Answer:- For any geostationary satellite time period
T = 24hours = 24x60x60s

=86400sec

Orbital velocity $v= 2\pi R/T$

Where R= distance of satellite from the earth. It is given as $R = R_E + h$
$F_c = mv^2/R$
$F_G = GM_e m/R^2$
$F_c = F_G$
$mv^2/R = GM_e m/R^2$
By simplifying,

$v^2 = GM_e/R$
$=4\pi^2 R^2/T^2 = GM_e/R$
$R^3 = GM(T^2/4\pi^2)$
Acceleration due to gravity $g = GM/R_E^2$
$GM = gR_E^2$
$R^3 = gR_E^2\, T^2/4\pi^2$
Therefore $R = [gR_E^2\, T^2/4\pi^2]^{1/3}$
By putting the values and calculating,

R= 42147Km
$R = R_E + h$
$h = R - R_E$
$=(42147 - 6.37 \times 10^3)$
h= 35777km

The height of the geostationary satellite from the surface of the earth is 35777km.

Polar Satellites
- These are low altitude satellites.This means they orbit around earth at lower heights.
- They orbit around the earth in North-South direction.Whereas earth is moving from East to West.
- A camera is fixed above this type of satellite so they can view small strips of earth.
- As earth also moves, so at each instance different types of stripes of earth can be viewed.
- Adjacent stripes of earth are viewed in subsequent orbits.
- They are useful in remote sensing, meteorology and environmental studies of the earth.

In the above image we can see that the orbit of polar satellites is from north to south direction.

Weightlessness

- Weightlessness is a condition of free fall, in which the effect of gravity is cancelled by the inertial (e.g., centrifugal) force resulting from orbital flight. There is no force of gravity acting on the objects.
- It is the condition in which body does not feel its weight at all.
- When an apple falls from a tree it won't feel its weight. This condition experienced by anybody while in free-fall is known as weightlessness.

Examples: -When we throw an object from the top of building, the object experiences free fall, that is the object is not under any force.This is weightlessness.

Weightlessness in the orbital motion of satellites

- In case of a satellite that is rotating around the earth.
- There is an acceleration which is acting towards the centre of the Earth.
- This acceleration is known as centripetal acceleration (a_c).
- There is also earth's acceleration which is balancing this centripetal acceleration.
  - $g=a_c$ they are equal in magnitude and they are balancing each other.
- Inside the satellites there is no acceleration which means everything is moving with uniform velocity.
- Inside an orbiting satellite weightlessness is experienced.

Thank You

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