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Introduction

- This lesson is focussed on how we stretch, compress and bend a body.
- We have seen that if we stretch a rubber and let it go it deforms but by applying the force, it again regains its original shape.
- If we take an aluminium rod and apply very strong external force on it, the shape of the rod might change but it does not regain its original position.
- We will take a look why some materials are elastic and why some are plastic.

Ball of rubber bands
Aluminium rod

Solids and their mechanical properties

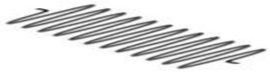
- Mechanical Properties of solids describe characteristics such as their strength and resistance to deformation.
- It describes about the ability of an object to withstand the stress applied to that object. Objects also resist changing their shape.
- For example:- Objects such as clay can be easily deformed so they have less resistance to deformation but objects like iron don't change their shapes easily. When heated they change their shapes which means they have very high resistance to deformation.

Clay can be moulded in the shape of an earthen pot.

Mechanical properties:-

1. **Elasticity**: - Elasticity is a property by virtue of which original shape is regained once the external force is removed.

- This means it tells us how much elastic a body is.
- For example:- A spring .If we stretch a spring it changes its shape and when the external force is removed spring comes back to its original position.

Spring , Rubber Band

2. Plasticity: - Plasticity is reverse of elasticity.

- Property means permanent deformation.
- The object never regains its original shape even when the external force is removed. These types of objects are called as plastics.
- For example:- Toys, Buckets made up of plastics.

Plastic bucket
Toy made of plastic.

3. Ductility: - Property of being drawn into thin wires or sheets.

- For Example: - Small chains of gold and silver.

Chain made of gold.

4. **Strength**: - Ability to withstand applied stress without failure.

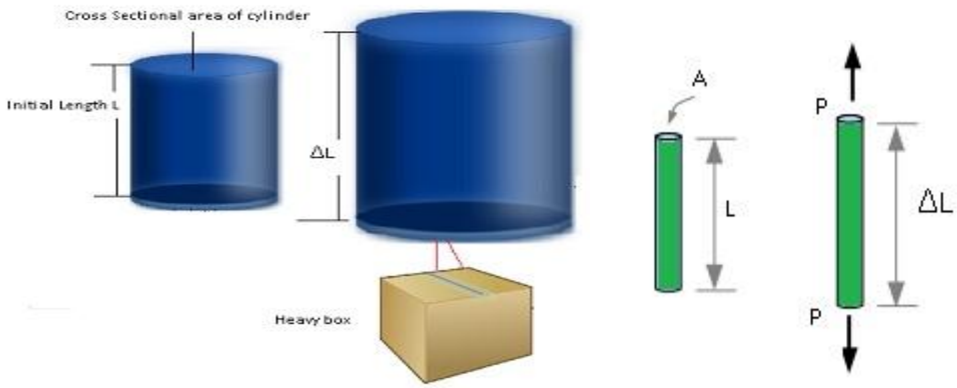
Stress:-

- Stress is the restoring force per unit area.
- Whenever we apply an external force on the body to change its shape there is a restoring force that develops in the body in the opposite direction.
- For example:-
- When we apply an external force to a rubber ball at the same instant of time some force develops in the ball which acts in the opposite direction.
- This opposite force which develops in the ball when an external force is applied is known as restoring force.
- Both the forces are equal in magnitude.
- Mathematically:-
- **Stress = F/A**
- Where F= restoring force develops in the body because of force we apply.
- A=area
- S.I. Unit :- N/m^2 or Pascal(Pa)
- Dimensional formula is $[ML^{-1}T^{-2}]$.

Types of Stress: **Longitudinal stress**

- Longitudinal stress is defined as restoring force per unit area when the force is applied to the cross-sectional area of the cylindrical body.

- Consider a cylinder which we have to deform. If we apply the force perpendicular to the cross-sectional area, there will be a restoring force that develops in the cylinder in the opposite direction.
- This restoring force per unit area is known as longitudinal stress.
- Experimentally we can observe the increase in length.
- If we tie a heavy object to the cylinder with the help of threads.
- Let Initial length of the cylinder is L.
- After it gets stretched its length increases by ΔL due to the stress.
- As there is change in the length therefore this type of stress is known as longitudinal stress.
- In the below figure if we attach a box to the cylinder, a force is applied on the cross-sectional area of cylinder due to which it gets stretched and as a result there is change in the length of the cylinder.



Problem:- A 14.5 kg mass, fastened to the end of a steel wire of unstretched length 1.0 m, is whirled in a vertical circle with an angular velocity of 2 rev/s at the bottom of the circle. The cross-sectional area of the wire is 0.065 cm^2. Calculate the elongation of the wire when the mass is at the lowest point of its path.

Answer:- Mass, m = 14.5 kg
Length of the steel wire, l = 1.0 m
Angular velocity, ω = 2 rev/s
Cross-sectional area of the wire, a = 0.065 cm^2
Let Δl be the elongation of the wire when the mass is at the lowest point of its path.
When the mass is placed at the position of the vertical circle, the total force on the mass is:
$F = mg + ml\omega^2$
$= 14.5 \times 9.8 + 14.5 \times 1 \times (2)^2 = 200.1$ N
Young's modulus =Stress/Strain
Y = (F/A)/ Δl/l
= (F l)/A Δl
Therefore Δl = F l/A Y
Young's modulus for steel = 2×10^{11} Pa
Therefore $\Delta l = 200.1/0.065\text{x}10^{-4}\text{x}2\text{x}10^{11} = 1539.23\text{x}10^{7}$
$=1.539\text{x}10^{-4}$m
Hence, the elongation of the wire is 1.539×10^{-4} m.

Types of Longitudinal Stress:-

1. Tensile Stress
2. Compressive Stress

Tensile Stress

- Tensile stress is a longitudinal stress when the length of the cylinder increases.

For example:-

- When the force is applied to both sides of the cylinder, the cylinder gets stretched. As a result there will be increase in its length.

Force is applied on both the sides as a result length of cylinder increases

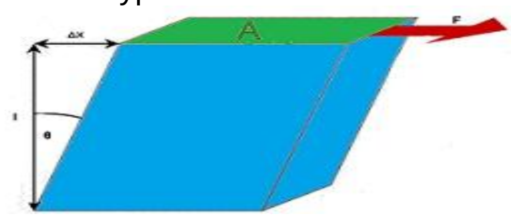
Compressive Stress

- Compressive stress is a longitudinal stress where the force is applied to compress the cylinder.

Compressing the cylinder

Tangential or Shearing Stress

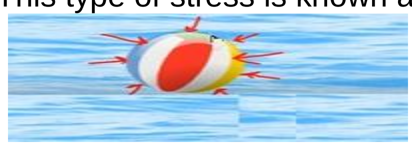
- Restoring force per unit area when the force applied is parallel to the cross sectional area of the body.
- Relative displacement occurs between the opposite faces of the body.
- For example:-
- Consider a cube. If we apply force parallel to the cross sectional area there will be movement which takes place between the opposite faces of the cube as they have relative motion with each other.
- This type of stress is known as tangential or shearing stress.



Hydraulic Stress

- Hydraulic stress is the restoring force per unit area when force is applied by a fluid on the body.
- For example:-

1. Consider a rubber ball and if it is dipped in the pond .Due to the pressure of water from all directions force acts on the ball as a result, the ball seems to be slightly contracted.
2. Because of the force exerted by the water there is restoring force which develops in the ball which is equal in magnitude to the force applied by the water but in opposite direction.
3. This type of stress is known as hydraulic stress.



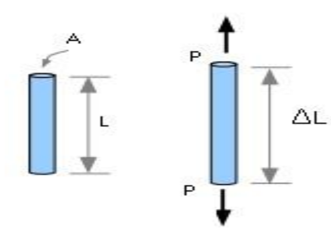
Ball under the water

Strain

- Strain is a measure of deformation representing the displacement between particles in the body to a reference length.
- It tells us how and what changes takes place when a body is subjected to strain.
- Mathematically:- **Strain** = **ΔL/L ,** where ΔL=change in length L= original length
- It is dimensionless quantity because it is a ratio of two quantities.
- For example: - If we have a metal beam and we apply force from both sides the shape of the metal beam will get deformed.
- This change in length or the deformation is known as Strain.

Types of Strain: Longitudinal Strain

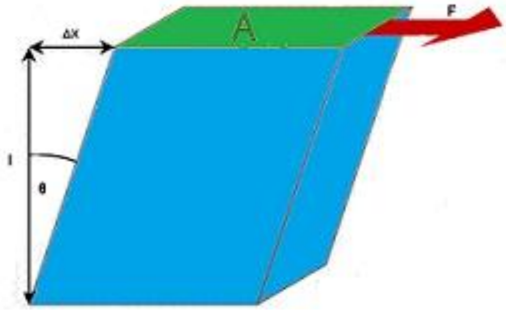
- Change in length to the original length of the body due to the longitudinal stress.
- If we apply longitudinal stress to a body either the body elongates or it compresses this change along the length of the body. This change in length is measured by Longitudinal Strain.
- Longitudinal Strain = ΔL/L
- Mathematically
- Consider a rod whose initial length is L after elongation length becomes L'.
- So the change in length is ΔL= L' – L
- So Strain= ΔL/L
- Strain occurs as a result of stress.



Shearing Strain

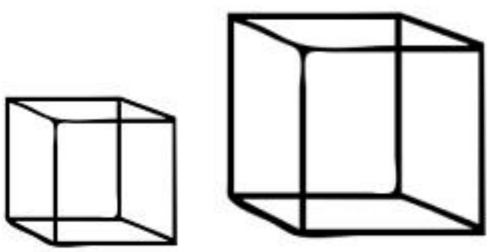
- Shearing strain is the measure of the relative displacement of the opposite faces of the body as a result of shearing stress.
- If we apply force parallel to the cross - sectional area because of which there was relative displacement between the opposite faces of the body.
- Shearing strain measures to what extent the two opposite faces got displaced relative to each other.
- Mathematically:-
- Consider a cube whose initial length was L which is at some position and when it gets displaced by an angle θ.
- Let the small relative displacement be x.
- **Shearing strain= x/L**

- In terms of tan θ,
- Shearing strain = tan θ = x/L
- tan θ is equal to θ (as θ is very small)
- Therefore, **x/L = θ**



Volume Strain

- Volume strain is defined as ratio of change in volume to the original volume as a result of the hydraulic stress.
- When the stress is applied by a fluid on a body there is change in the volume of body without changing the shape of the body.
- **Volume strain = ΔV/V**
- For example:-
- Consider a ball initially at volume V.



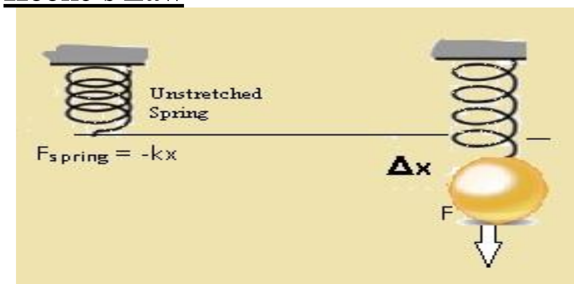
- Because of hydraulic stress there is change in volume V'
- Therefore, Change in the volume ΔV= V' – V
- Conclusion: - Deformation is measured using strain.
- Consider a cube whose initial volume is V.
- When the cube is subjected to stress there will be a change in the volume but the shape will not change.

Problem:- Four identical hollow cylindrical columns of mild steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and compressional strain of each column.

Answer:
Mass of the big structure, M = 50,000 kg
Inner radius of the column, r = 30 cm = 0.3 m
Outer radius of the column, R = 60 cm = 0.6 m
Young's modulus of steel, Y = 2 × 1011 Pa
Total force exerted, F = Mg = 50000 × 9.8 N
Stress = Force exerted on a single column = 122500 N
Young's modulus, Y = $Stress/Strain$

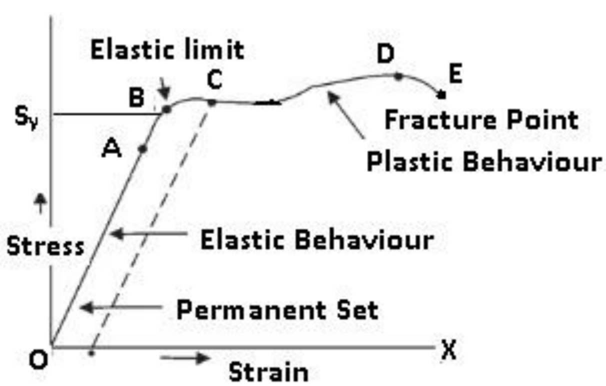
Where,
Area, A = π (R^2 – r^2) = π (($(0.6)^2$ – $(0.3)^2$)
Strain = 122500/ (π [$0.6)^2$ – $(0.3)^2$] x $2x10^{11}$)
= 7.22×10^{-7}
Hence, the compressional strain of each column is 7.22×10^{-7}

Hooke's Law



- Robert Hooke was the scientist who gave Hooke's law.
- Hooke's law states that within the elastic limit, stress developed is directly proportional to the strain produced in a body.
- Consider a scenario where we apply external force to the body. As a result stress develops in the body due to this stress there will be a strain produced in the body which implies that there will be some deformation in the body.
- Because of stress, strain is produced.
- According to Hooke's law, if strain increases the stress will increase and vice-versa.
- The Hooke's law is applicable to all elastic substances.
- It does not apply to plastic deformation.
- Mathematically :
- stress ∝ strain
- stress = k × strain
- Where k is the proportionality constant and is known as modulus of elasticity.

Stress- Strain Curve



- It is a curve between stress and strain.
- A graph is plotted between the stress (which is equal in magnitude to the applied force per unit area) and the strain produced.

- The graph helps us to understand how a given material deforms with increasing loads.
- The curve between O and A, is a straight line. This means stress is directly ∝ to strain. In this region Hooke's Law is applicable.
- In this region the material behaves like an elastic body.
- In the region from A to B, stress and strain are not directly ∝. But still the material returns to its original dimension after the force is removed. They exhibit elastic properties.
- The point B in the curve is known as **yield point** (also known as elastic limit) which means till this point the material will be elastic in behaviour and the stress corresponding to point B is known as yield strength (S_y) of the material.
- The region between O and B is called as **Elastic region.**
- From point B to point D we can see that strain increases rapidly even for small change in stress.
- Even if we remove the force the material does not come back to its original position. At this point stress is zero but strain is not zero as body has changed its shape.
- The material has undergone **plastic deformation**.
- The material is said to be **permanent set**.
- The point D on the graph is known as ultimate tensile strength (S_u) of the material.
- From D to E we can see that stress decreases even if strain increases.
- Finally at point E fracture occurs. This means the body breaks.
- **Conclusion**:-
- An object is brittle if D and E are very close. This means fracture point is near to tensile strength.
- For example:-Glass which is brittle.


Glass Jar
If Glass jar is dropped it will easily break into pieces.

An object is ductile if D and E are very far apart from each other. This means fracture point is far away from tensile strength.

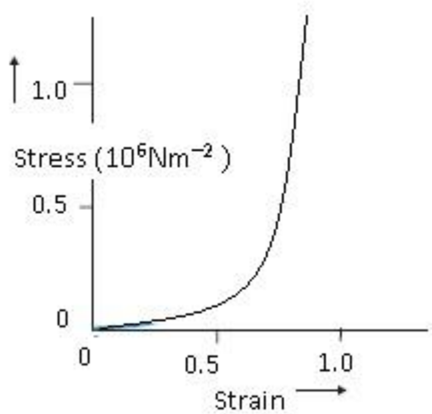
For example:-Metals, Gold and silver etc.

Elastic substances like rubber have larger elastic region.

For example:-spring, catapult, tissue of aorta etc.

Below is the graph between stress-strain for the elastic tissue of aorta, present in the heart.

We can see from the graph even though elastic region is very large, the material does not obey Hooke's law over most of the region.



Elastic Modulus

- Elastic modulus is ratio of stress and strain.
- Elastic modulus is a characteristic value of each material. This means gold will have specific value of elastic modulus and rubber will have specific value of elastic modulus etc.
- k=Stress/Strain where k= Elastic modulus.

Problem: - A piece of copper having a rectangular cross-section of 15.2 mm × 19.1 mm is pulled in tension with 44,500 N forces, producing only elastic deformation. Calculate the resulting strain?

Answer:

Length of the piece of copper, $l = 19.1\ mm = 19.1 \times 10^{-3}\ m$

Breadth of the piece of copper, $b = 15.2\ mm = 15.2 \times 10^{-3}\ m$

Area of the copper piece:

$A = l \times b = 19.1 \times 10^{-3} \times 15.2 \times 10^{-3} = 2.9 \times 10^{-4}\ m^2$

Tension force applied on the piece of copper, F = 44500 N

Modulus of elasticity of copper, $\eta = 42 \times 10^9\ N/m^2$

Modulus of elasticity, η = Stress/Strain

= (F/A)/Strain

Strain = F/ (A η) $=44500/2.9x10^{-4}x42x10^9$

$= 3.65 \times 10^{-3}$

Types of Elastic Modulus

1. Young's Modulus
2. Shear Modulus
3. Bulk Modulus

Young's Modulus

- Young's modulus is derived from the name of the scientist who defined it.
- It is the ratio of longitudinal stress to longitudinal strain.
- It is denoted by Y.
- Mathematically:
- Y= longitudinal stress/ longitudinal strain = σ/ ε
- = (F/A)/ (ΔL/L)
- **Y=FL/ΔL**
- If Young's modulus is more, to produce a small change in length more force required.
- S.I. Unit is N m^{-2} or Pascal (Pa).
- Metals have comparatively greater Young's Modulus. To change the length of metals, greater force is required.

Problem: - A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a load, the net

elongation is found to be 0.70 mm. Obtain the load applied.

Answer:- The copper and steel wires are under a tensile stress because they have the same tension (equal to the load W) and the same area of cross-section A.
Stress = strain × Young's modulus. Therefore

$W/A = Y_c \times (\Delta L_c/L_c) = Y_s \times (\Delta L_s/L_s)$ where
The subscripts c and s refer to copper and stainless steel respectively. Or,

$\Delta L_c /\Delta L_s = (Y_s/Y_c) \times (L_c /L_s)$
Given Lc = 2.2 m, Ls = 1.6 m, Yc = 1.1 × 1011 Nm^{-2}, and $Y_s = 2.0 \times 10^{11}$ Nm^{-2}.
$\Delta L_c/\Delta L_s = (2.0 \times 10^{11}/1.1 \times 10^{11}) \times (2.2/1.6) = 2.5$.
The total elongation is given to be

$\Delta L_c + \Delta L_s = 7.0 \times 10^{-4}$ m
Solving the above equations,

$\Delta L_c = 5.0 \times 10^{-4}$ m, and $\Delta L_s = 2.0 \times 10^{-4}$ m.
Therefore

$W = (A \times Y_c \times \Delta L_c)/L_c$
$= \pi (1.5 \times 10^{-3})^2 \times [(5.0 \times 10^{-4} \times 1.1 \times 10^{11})/2.2]$
$= 1.8 \times 10^2$ N

Problem: A structural steel rod has a radius of 10 mm and a length of 1.0 m. A 100 kN force stretches it along its length. Calculate (a) stress, (b) elongation, and (c) strain on the rod. Young's modulus, of structural steel is 2.0×10^{11} N m^{-2}.
Answer: We assume that the rod is held by a clamp at one end, and the force F is applied at the other end, parallel to the length of the rod.
Then the stress on the rod is given by

Stress =F/A = $F/\pi r^2$
=100 10^3 N/3.14 10^2 m^2
= 3.18×10^8 N m^{-2}
The elongation,

ΔL = (F/A) Y

= (3.18 10^8 N m^{-2} 1m)/ (2 10^{11} N m^{-2})
= 1.59×10^{-3} m
= 1.59 mm

The strain is given by

Strain = ΔL/L

= (1.59×10^{-3} m)/ (1m)
= 1.59×10^{-3}
= 0.16 %

Young's Modulus: Application

- In industrial constructions steel is preferred over copper. The reason behind this is steel is more elastic than copper.
- If there is slight deformation in steel due to contraction and expansion it will come back to its original position.
- Steel is preferred over copper to construct bridges.

Problem:- A steel wire of length 4.7 m and cross-sectional area 3.0×10^{-5} m^2 stretches by the same amount as a copper wire of length 3.5 m and cross-sectional area of $4.0 \times 10^{-5} m^2$, under a given load. What is the ratio of the Young's modulus of steel to that of copper?
Answer:- Length of the steel wire, L_1 = 4.7 m
Area of cross-section of the steel wire, $A_1 = 3.0 \times 10^{-5} m^2$
Length of the copper wire, L_2 = 3.5 m
Area of cross-section of the copper wire, $A_2 = 4.0 \times 10^{-5} m^2$
Change in length = $\Delta L_1 = \Delta L_2 = \Delta L$
Force applied in both the cases = F

Young's modulus of the steel wire:

$Y_1 = (F_1/A_1)(L_1/\Delta L)$
$= F \times 4.7/(3.0 \times 10^{-5} \times \Delta L)$ … (i)

Young's modulus of the copper wire:

$Y_2 = (F_2/A_2)(L_2/\Delta L)$
$= F \times 3.5/(4.0 \times 10^{-5} \times \Delta L)$ … (ii)

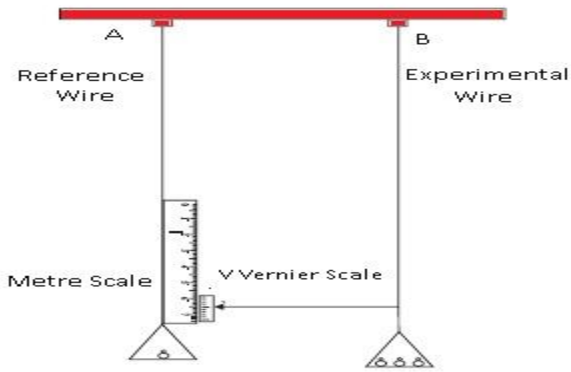
Dividing (i) by (ii), we get:

$Y_1/Y_2 = (4.7 \times 4.0 \times 10^{-5})/(3.0 \times 10^{-5} \times 3.5)$
$= 1.79:1$

The ratio of Young's modulus of steel to that of copper is 1.79: 1.

Determination of Young's Modulus of the material of the wire

Experimental set up:-



- Two strings were hung from a support and two pans were attached to both the strings.
- Weights are kept on both the pans.
- When the number of weights in second pan was increased, the string got stretched and moved in downward direction.
- The change in length was measured by the metre scale which was kept on reference wire.
- Using this experiment, the Young's modulus value was calculated
- Y= longitudinal stress/ longitudinal strain = σ/ε
- $= (F/A)/(\Delta L/L)$
- Where original length = L and ΔL = change in length, F=mg (acting downwards) and A (area of cross-section of wire) = πr^2
- $= (mg/\pi r^2)/(\Delta L/L)$
- $\mathbf{Y = mgL/\pi r^2 \Delta L}$
- This is the way to calculate the Young's modulus.

Problem:- Read the following two statements below carefully and state, with reasons, if it is true or false.

1. a) The Young's modulus of rubber is greater than that of steel;
2. b) The stretching of a coil is determined by its shear modulus.

Answer:
(a) False
(b) True
For a given stress, the strain in rubber is more than it is in steel.
Young's modulus, Y = $Stress/Strain$
For a constant stress: $Y \propto 1/Strain$
Hence, Young's modulus for rubber is less than it is for steel.
Shear modulus is the ratio of the applied stress to the change in the shape of a body. The stretching of a coil changes its shape. Hence, shear modulus of elasticity is involved in this process.

Problem:- A rigid bar of mass 15 kg is supported symmetrically by three wires each 2.0 m long. Those at each end are of copper and the middle one is of iron. Determine the ratio of their diameters if each is to have the same tension.
Answer:- The tension force acting on each wire is the same.
Thus, the extension in each case is the same. Since the wires are of the same length, the strain will also be the same.
The relation for Young's modulus is given as:
$Y = Stress/Strain = (F/A)/Strain = (4F/\pi d^2)/Strain$ Equation (i)
Where,
F = Tension force
A = Area of cross-section
D=Diameter of the wire
From equation (i) $Y \propto 1/d^2$
Young's Modulus for iron, $Y_1 = 190\times10^9$ Pa
Diameter of the iron wire $= d_1$
Young's Modulus for copper, $Y_2 = 110\times10^9$ Pa
Diameter of the copper wire $= d_2$
Therefore the ratios of their diameters are given as:
$d_1/d_2 = \sqrt{Y_1/Y_2} = \sqrt{190\times10^9/110\times10^9} = \sqrt{19/11} = 1.31:1$

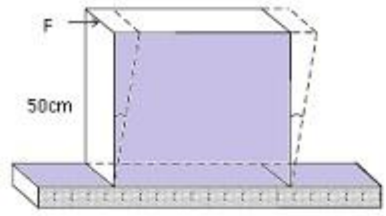
Shear Modulus (Modulus of Rigidity)
- Shear modulus is defined as shearing stress to shearing strain.
- It is also known as Modulus of Rigidity.
- It is denoted by 'G'.
- S.I. Unit: N/m^2 or Pascal(Pa)
- Mathematically
- G=shearing stress/shearing strain = $(F/A)/(\Delta x/L) = FL/A\,\Delta x$
- By the definition of shearing strain $1/\theta = (L/\Delta x)$
- $G = F/A\,\theta$

Relation between Young's Modulus and Shear Modulus
- Shear modulus is less than Young's modulus.
- For most materials **G** = **Y/3**.

Problem:- A box shaped piece of wax has a top area of $10cm^2$ and height of 2cm.When a shearing force of 0.5N is applied to the upper surface, displaces 4mm relative to the bottom surface. What are the shearing stress, shearing strain and shear modulus for wax?
Answer:- Area = $10cm^2$, height =2cm, (displacement) x=4mm
1. Shearing stress = tangential force/Area
$= 0.5/10\times10^{-4} = 500Pa$

2. Shearing strain = small displacement/initial length = x/L
   $= (4x10^{-3})/ (2x10^{-2}) = 0.2$
3. Shear Modulus = Shearing stress/ Shearing strain
   =500/0.2 = 2500 Pa.

Problem:- A square lead slab of side 50 cm and thickness 10 cm is subject to a shearing force (on its narrow face) of 9.0×10^4 N. The lower edge is riveted to the floor. How much will the upper edge are displaced?



Answer: - The lead slab is fixed and the force is applied parallel to the narrow face as shown in Fig. The area of the face parallel to which this force is applied is
A = 50 cm × 10 cm

= 0.5 m × 0.1 m

$= 0.05\ m^2$
Therefore, the stress applied is

$= (9.4 \times 10^4\ N/0.05\ m^2)$
$= 1.80 \times 10^6 Nm^{-2}$
We know that shearing strain = (Δx/L) = Stress /G. Therefore the displacement Δx = (Stress × L)/G = $(1.8 \times 10^6\ N\ m^{-2} \times 0.5m)/ (5.6 \times 109\ N\ m^{-2})$
$= 1.6 \times 10^{-4}$ m = 0.16 mm

Problem:- The edge of an aluminium cube is 10 cm long. One face of the cube is firmly fixed to a vertical wall. A mass of 100 kg is then attached to the opposite face of the cube. The shear modulus of aluminium is 25 GPa. What is the vertical deflection of this face?
Answer:
Edge of the aluminium cube, L = 10 cm = 0.1 m

The mass attached to the cube, m = 100 kg

Shear modulus (η) of aluminium = 25 GPa = 25×10^9 Pa
Shear modulus, η = Shear stress/Shear Strain = (F/A)/L/ ΔL

Where,

F = Applied force = mg = 100 × 9.8 = 980 N

A = Area of one of the faces of the cube = 0.1 × 0.1 = $0.01\ m^2$
ΔL = Vertical deflection of the cube

ΔL = FL/A η = $(980x0.1)/10\text{-}2x\ (25x10^9)$
$= 3.92 \times 10^{-7}$ m
The vertical deflection of this face of the cube is 3.92×10^{-7} m.

Bulk Modulus

- Bulk modulus is the ratio of hydraulic stress to the corresponding hydraulic strain.
- Denoted by 'B'
- B = -p/(ΔV/V)
- Where p =hydraulic stress, ΔV/V = hydraulic strain

(-) ive signs show that the increase in pressure results in decrease in volume.

- S.I. Unit :- N/m^2 or Pascal(Pa)
- B(solids) > B(liquids) >B(gases)

Compressibility

- Compressibility is the measure of compression of a substance.
- Reciprocal of bulk modulus is termed as 'Compressibility'.
- Mathematically:
- k=1/B = - (1/p) (ΔV/V)
- It is denoted by 'k'.
- k(solids)<k(liquids)<k(gases)

Problem:-The average depth of Indian Ocean is about 3000 m.
Calculate the fractional compression, ΔV/V, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2×10^9 $N\ m^{-2}$. (Take g = 10 $m\ s^{-2}$)
Answer: - The pressure exerted by a 3000 m column of water on the bottom layer
p = hρ g = 3000 m × 1000 kg m^{-3} × 10 m s^{-2}
= 3 × 107 kg m^{-1} s^{-2}
= 3 × 107 N m^{-2}
Fractional compression ΔV/V, is

ΔV/V = stress/B = (3 × 107 N m^{-2})/ (2.2×10^9 N m^{-2})
= 1.36×10^{-2} or 1.36 %

Problem: The bulk modulus for water is 2.1GPa.Calculate the contraction in volume of 200ml of water is subjected to a pressure of 2MPa.
Answer:-
B=2.1GPa = 2.1 $x10^9$Pa.
V=200ml = 2$x10^{-6}$ml.
P=2MPa = 200$x10^6$ Pa.
B=- (1/p) (ΔV/V) = ΔV = pV/B

= (2$x10^6$x200$x10^{-6}$)/ = 2.1 $x10^9$
B=0.19ml

Problem: - Compute the bulk modulus of water from the following data: Initial volume = 100.0 litre, Pressure increase = 100.0 atm (1 atm = 1.013×10^5 Pa), Final volume = 100.5 litre. Compare the bulk modulus of water with that of air (at constant temperature). Explain in simple terms why the ratio is so large.

Answer:
Initial volume, V_1 = 100.0l = 100.0 × 10^{-3} m^3
Final volume, V_2 = 100.5 l = 100.5 ×10^{-3} m^3
Increase in volume, ΔV = $V_2 - V_1$ = 0.5 × 10^{-3} m^3
Increase in pressure, Δp = 100.0 atm = 100 × 1.013 × 10^5 Pa
Bulk Modulus = Δp / ΔV/ V_1 = Δp x V_1/ ΔV
= (100x1.013 × 10^5x100x10^{-3})/0.5x10^{-3}
=2.026x10^6 Pa
Bulk modulus of air= 1.0x10^5Pa
Therefore,

Bulk modulus of water/ Bulk modulus of air

=2.026x10^6/1.0x10^5 =2.026x10^4
This ratio is very high because air is more compressible than water.

Problem:-
What is the density of water at a depth where pressure is 80.0 atm, given that its density at the surface is 1.03 × 10^3 kg m^{-3}?
Answer:-
Let the given depth be h.

Pressure at the given depth, p = 80.0 atm = 80 × 1.01 × 10^5 Pa
Density of water at the surface, ρ_1 = 1.03 × 10^3 kg m^{-3}
Let ρ_2 be the density of water at the depth h.
Let V_1 be the volume of water of mass m at the surface.
Let V_2 be the volume of water of mass m at the depth h.
Let ΔV be the change in volume.

ΔV = V_1 - V_2
=m (1/ ρ_1 – 1/ ρ_2)
Therefore, Volumetric strain = ΔV/ V_1
=m (1/ ρ_1 – 1/ ρ_2) x ρ_1/m
Therefore, ΔV/ V_1 = (1- ρ_1/ ρ_2) … (i)
Bulk modulus, B = p V_1/ ΔV
ΔV/ V_1 = p/B
Compressibility of water =1/B =45.8x10^{-11} Pa^{-1}
Therefore, ΔV/ V_1= 80x1.013x10^5x45.8x10^{-11} = 3.71x10^{-3} … (ii)
For equations (i) and (ii), we get:

1- ρ_1 / ρ_2 = 3.71 x10^{-3}
ρ_2 = 1.03x10^3/(1-(3.71x10^{-3})
=1.034x10^3kgm^{-3}
Therefore, the density of water at the given depth (h) is 1.034 × 10^3 kg m^{-3}.

Problem: - Compute the fractional change in volume of a glass slab, when subjected to a hydraulic pressure of 10 atm.
Answer:-
Hydraulic pressure exerted on the glass slab, p = 10 atm = 10 × 1.013 × 10^5 Pa
Bulk modulus of glass, B = 37 × 10^9 Nm^{-2}
Bulk modulus, B= p/ (ΔV/V)

Where,

ΔV/V = Fractional change in volume

Therefore,

ΔV/ V = p/B

=10x1.013x10^5
=2.73x10^{-5}
Hence, the fractional change in the volume of the glass slab is 2.73 × 10^{-5}.

Problem: - How much should the pressure on a litre of water is changed to compress it by 0.10%?
Answer:
Volume of water, V = 1 L
It is given that water is to be compressed by 0.10%.
Fractional change = ΔV/V =0.1/100x1 = 10^{-3}
Bulk modulus, B= ρ/ ΔV/V
p=B x ΔV/V
Bulk Modulus of water, B = 2.2x10^{9}Nm^{-2}
p=2.2x10^{9}x10^{-3}
=2.2x106Nm^{-2}
Therefore, the pressure on water should be 2.2 ×10^6 Nm^{-2}.

Problem: - The average depth of Indian Ocean is about 3000 m. Calculate the fractional compression, ΔV/V, of water at the bottom of the ocean, given that the bulk modulus of water is 2.2 × 10^9 N m^{-2}. (Take g = 10 m s^{-2})
Answer:- The pressure exerted by a 3000 m column of water on the bottom layer
p = hρ g = 3000 m × 1000 kg m^{-3} × 10 m s^{-2}
= 3 × 10^7 kg m^{-1} s^{-2}
= 3 × 10^7 N m^{-2}
Fractional compression ΔV/V, is

ΔV/V = stress/B = (3 × 10^7 N m^{-2})/ (2.2 × 109 N m^{-2})
= 1.36 × 10^{-2} or 1.36 %

Thank You

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Notes provided by

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